Whitepaper on the event rates and detectability of extreme-mass-ratio inspirals with LISA

Michele Vallisneri
Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA 91109
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I survey the current effort in estimating the rates of compact-body inspirals into supermassive black holes, and in evaluating the feasibility of detecting gravity waves from such events using LISA.

History

Changes from version 1.1: updated discussion of capture rates (Sec. III) with Bender's argument about gradual approach to coalescence, and with Freitag's recent results about light main-sequence stars; clarified the significance of $N_{\rm templates}$ (Sec. IV); updated status of numerical implementation of kludged templates (Sec. IV B); updated discussion of counting for AK templates, including effect of angle-maximized detection statistics (Sec. IV C 1), and for NK templates (Sec. IV C 1); updated final estimate of detection rates (Sec. V); updated references.
Changes from version 1.0: updated discussion of expected optimal S/Ns (Sec. III); discussion of implementation of numerical kludges (Secs. IV B 2, 3); discussion of progress and prospects in the counting of AK templates (Sec. IV C 1) and NK templates (Sec. IV C 2); *new estimate of detection rates* (Sec. V).

I. INTRODUCTION

There is considerable (and increasing) evidence that the centers of most galaxies contain massive black holes (MBH) with masses between 10^6 and 10^9 solar masses [1]. The MBHs sit at the center of very densely populated, relaxed stellar clusters; the stars in the clusters will occasionally be scattered into very eccentric, relativistic *capture orbits*, which are shrunk by energy loss to gravity-wave emission on a timescale shorter than the time needed to scatter them back into more loosely bound orbits.[1] Main-sequence stars will be disrupted by the MBH tidal field at typical capture-orbit pericenters (see however Sec. III), so only evolved compact objects such as white dwarfs (WDs), neutron stars (NSs), and solar-mass black holes (SBHs) will generally be found in such orbits. Often the compact objects (henceforth CBs) will plunge rapidly into the MBH, emitting gravitational radiation over a very short time; the resulting gravity waves are not observable. Other times, however, the approach will be more gradual, and the gravity waves will be emitted over longer times at characteristic frequencies of order 10^{-4}–10^{-2} Hz, with a good chance of being detected by LISA [3].

The scientific payoff for observing CB-inspiral waves would be remarkable (see Sec. II), so in December 2001 the LIST recommended setting the minimum of the LISA noise curve by requiring that CB-inspiral waves be detected in sufficient numbers and with sufficient signal-to-noise ratios. Two elements play crucially into the sensitivity level required for this purpose: the rate of gravitational captures (see Sec. III), which determines the number of events detectable, in principle, within the duration of the mission; and the tools available to detect and analyze the waveforms (see Sec. IV), which determine the detection efficiency. Work is currently underway to improve our understanding of both elements, with the dual purpose of firming up the LISA Science Requirements and of providing a roadmap for the necessary theoretical and computational advances before launch time. The purpose of this whitepaper is to summarize the current status of this work, with a particular focus on the partial objectives that can be considered as acquired, and on the most promising directions of improvement.

II. MAIN SCIENTIFIC PAYOFFS

Census of astrophysical parameters. The successful detection of CB-inspiral waves from a population of CBs and MBHs would yield distributions for MBH parameters (mass, spin, distance), CB orbital parameters (eccentricity and inclination), all with error $\lesssim$ a few percent out to redshifts beyond one [4, 5]. Of course, the detections

[1] This timescale is a fraction of the relaxation timescale $T_{\rm rel}$ [2].

would also provide information about capture rates. All these data would in turn allow insight into the evolution of MBHs and into the characteristics of the surrounding star clusters.

No-hair theorems. CB-inspiral waves contain information about the higher mass and current multipoles of the central massive object, allowing a comparison with the values predicted by the Kerr metric [4, 6]. Different results would either falsify the predictions of general relativity in the strong-gravity regime, or indicate the presence an exotic central massive object (such as a solitonic star, a naked singularity, or some kind of composite stellar object).

Energy extraction from MBHs. CB-inspiral waves would also contain detailed information about the tidal extraction of energy from the MBH to the CB [4].

III. CAPTURE RATES

Several authors [2, 3, 7–10] have computed the rates of gravitational captures of CBs by MBHs. The results are discordant, with a strong dependence on the underlying models of the galactic nuclei and on the analytic approximations used to treat the various physical processes involved. Recently, Freitag and Benz have developed a computer code targeted to follow the evolution of the star clusters around MBHs over times as long as 10^{10} yrs [11, 12]. The code is based on a Monte Carlo method *à la* Hénon, which follows the relaxation-driven evolution of the cluster in phase space without integrating directly the equations of motion for the stars; the basic dynamical object of the simulation (the *superstar*) represents ~ 100 stars of the same type, mass, and orbital parameters. Using this code, Freitag has estimated capture rates of a few 10^{-7}/yr for WDs and $\sim 5 \times 10^{-8}$/yr for NSs and SBHs in a galaxy similar to our own [13]. Freitag cautions that these numbers are not very firm because in his simulation only few superstars can be allocated to evolved objects while maintaining the correct proportions of stellar species within the cluster.

Phinney [14] argues that detection should be dominated by $\sim 10M_\odot$ SBHs spiraling into $10^6 M_\odot$ MBHs. Black-hole inspirals are preferred for two reasons: first, although SBHs represent only $\sim 1\%$ of the population of evolved stars, the stellar cusp will be disproportionately richer in these heavier objects because of mass segregation induced by dynamical friction; second, because their mass is higher, SBHs will yield inspiral waves with higher S/N (see Sec. IV) than WDs or NSs. Moreover, according to simulations by Bender and Hils [15], low-mass CBs ($\sim 1M_\odot$) interact more strongly with other stars in the cusp and plunge in more rapidly than heavier objects ($\sim 10M_\odot$), so the fraction of gradual approaches to coalescence (which produce produce detectable gravitational waves) is ~ 100 times smaller.

As for the mass of the central object, MBHs at the lower end of the observed mass range are preferred because galaxies similar to our own have larger relaxed stellar cusps, and because CB-inspiral waves enter the LISA frequency range much earlier in the evolution of the inspiral. Using Freitag's capture rates [13], Phinney estimates 1–10 inspirals per year out to one Gpc, with optimal signal-to-noise ratios (see below) from 60 to 200 over one year of integration. These values were computed by Finn and Thorne [5] for a $10M_\odot$ CB inspiraling into a $10^6 M_\odot$ MBH on a quasicircular inspiral orbit at a distance of one Gpc, using a sky-averaged noise curve based on the Pre-Phase A noise budget [16]. The lower bound corresponds to the case of no MBH spin; the upper bound to quasimaximal spin ($J = 0.999\, M^2$).

Finally, Freitag has recently argued [17] that main-sequence stars with mass $\lesssim 0.1M_\odot$ in capture orbits in galactic cusps similar to our own could resist tidal disruption and provide observable gravitational waves. He estimates that our Galaxy could have 0.5–5 such sources with S/N > 10.

Prospects: Phinney suggests that Freitag's simulations could be improved vastly by introducing a relativistic treatment of the capture process, and by integrating directly the equations of motion of the compact objects on the background of the relaxed cluster. As a first step, Gair, Kennefick, and Larson have computed the relativistic energy lost in a close-encounter parabolic orbit [18]. Cutler and Barack [19] have reevaluated gravity-wave emission for highly eccentric inspirals: while the total S/N changes little with respect to Finn and Thorne's results, the main contribution to gravity-wave emission comes from higher harmonics of the orbital frequency, so the waves should fall into a more favorable region of the LISA noise curve with respect to the confusion background of galactic binaries.

IV. FEASIBILITY OF DATA ANALYSIS

The detection of signals of known shape in a noisy background is best seen as a probabilistic problem: given a stretch of detector output and a theoretical characterization of the expected signals, we wish to evaluate the probability that the signal is actually present in the data stream, and that it is not being simulated (*false detection*) or hidden (*false dismissal*) by a particular instantiation of noise; furthermore, we wish to determine the physical parameters (e.g., the masses of binary constituents) that are most likely to have originated the observed signal, and to know the accuracy of this determination.

The optimal technique[2] to do so is *matched filtering*, whereby the measured signal is *correlated*[3] with a bank of theoretical *templates*, which represent the expected signal for a variety of physical parameters; the higher the correlation, the lower the probability of false alarm [20]. In fact, the criterion of successful detection for a given false-alarm probability $P_{\rm false}$ amounts to requiring that the measured correlation be higher than a *threshold* $({\rm S/N})^*_{\rm opt}$, given by[4]

$$P_{\rm false} = N_{\rm templates} \exp\left\{-\frac{[({\rm S/N})^*_{\rm opt}]^2}{2}\right\}, \tag{1}$$

where the *optimal signal-to-noise ratio* $({\rm S/N})_{\rm opt}$ is defined as the noise-weighted correlation of the signal with itself (i.e., with a perfect template), expressed in terms of the Fourier transform $s(f)$ of the signal by

$$[({\rm S/N})_{\rm opt}]^2 = 4\,{\rm Re} \int_0^\infty \frac{\tilde{s}^*(f)\tilde{s}(f)}{S_n(f)} df, \tag{2}$$

with $S_n(f)$ the one-sided noise power spectral density. The optimal signal-to-noise ratio is inversely proportional to the distance to the source, and it is an index of the total signal power available for detection, normalized by noise power. Note that Eq. (1) includes the total number of theoretical templates $N_{\rm templates}$: this is because the correlation of detector output with each template is counted as an independent statistical trial subject to the possibility of false alarm. However, Eq. (1) overestimates $P_{\rm false}$ when nearby templates (i.e., templates with close parameters) have high correlations, as it is desirable to avoid signals falling, as it were, between the cracks; in this situation the statistical trials are not truly independent.

So far we have introduced a certain amount of technical detail to justify two conclusions, which follow from the basic fact that the S/N detection threshold determines the maximum distance to which we can detect sources of a given type:

1. It is important to build theoretical templates that resemble the actual physical signals as closely as possible. Otherwise, the effective S/N will be lower than $({\rm S/N})_{\rm opt}$; consequently, the detection distance will decrease by the ratio $({\rm S/N})_{\rm eff}/({\rm S/N})_{\rm opt}$, and the rate of detections (proportional to the detection *volume*) will decrease by the cube of that ratio.

2. It is also important to limit the number of templates required to cover the region of physically plausible source parameters, because the detection threshold increases (weakly) with $N_{\rm templates}$, and (perhaps more importantly) because $N_{\rm templates}$ sets the amount of computational power needed to perform a search. This number can be estimated by computing the spacings of the template parameters needed to lay down a discrete *grid* of templates that achieve a minimum[5] $({\rm S/N})_{\rm eff}/({\rm S/N})_{\rm opt}$ for a physical signal anywhere within the grid.

The rest of this section is concerned with the ongoing effort to develop reliable templates, with the evaluation of the number of templates (and consequently of the detection threshold) required for a statistically convincing detection, and with the ensuing predictions for the rate of detections.

A. CB inspiral waveforms: status of theory

The very small ratio μ/M of CB mass to MBH mass (typically $\mu/M \sim 10^{-4}$–10^{-6}) suggests a description of inspiral orbits as quasiadiabatic sequences of geodesics in the MBH geometry (typically Kerr), where the integrals of the motion (for the Kerr geometry, these are the energy E, the angular momentum L_z along the MBH spin, and Carter's constant Q, which determines the inclination ι of the orbit) evolve while energy and angular momentum are carried away by gravitational radiation. The timescale for this evolution is much longer than the orbital period, so the

[2] *Optimal* in the technical sense that, among linear techniques, matched filtering yields the lowest false-dismissal probability for a given false-alarm probability.
[3] The correlation is actually weighted to emphasize the parts of the spectrum less affected by instrument noise [see Eq. (2)].
[4] This formula for the threshold is correct for the *phase-maximized* signal-template correlation [20] under the assumption of Gaussian noise.
[5] Here the reduction from the optimal S/N happens because the values of the parameters represented in the grid do not match exactly the actual source parameters; this reduction is distinct in origin from that caused by the approximation or incompleteness of the theoretical model. The two reductions are quantified, respectively, by the *minimum match* and the *fitting factor* [20].

gravitational perturbations to the MBH geometry caused by the CB can be computed in an orbit-averaged fashion using the Teukolsky–Sasaki–Nakamura formalism, yielding the fluxes at infinity (and therefore the rates of change) of E and L_z [21]. The evolution of Q is a different matter, and it is known exactly only for the special cases of equatorial orbits (where $Q = 0$) [22] and circular, inclined orbits (where Q evolves in a such a way that $e = \dot{e} = 0$) [21, 23]. At the moment, $\dot{Q}$ can be computed for generic orbits only using a weak-field quadrupole-moment formula [24] that is certainly inadequate for the close orbital separations reached in the most eccentric CB inspirals. A method of wider applicability, formulated in terms of the gravitational *self force* acting on the CB, is currently the subject of intense investigation: Barack, Ori [25], and (separately) Mino [26] have provided expressions for the self force in the Kerr geometry; the next step is then to double-check the expressions, and to verify whether they are suitable for numerical use.

With all probability, the completion of this research program will be a prerequisite to develop accurate template banks that can be used confidently with the LISA data stream. However, at this time it is necessary to start planning ahead for CB-inspiral data analysis, and to adjust instrument sensitivity to the level where the projected capture rates, detection distance, and detection efficiency promise to deliver the expected scientific payoffs. These assessments require a working model of CB signals to evaluate $N_{\mathrm{templates}}$, and therefore $(\mathrm{S/N})^*_{\mathrm{opt}}$. The strategy is then to use multiple families of simplified *kludge* waveforms that are perhaps far from realistic signals, but that include all the basic physical effects present in CB inspirals. Hopefully such waveforms will capture the functional complexity (if not the real shapes) of the true waveforms, which is all we need to estimate $N_{\mathrm{templates}}$.

Within the quasiadiabatic framework, a suitable family of kludge waveforms is obtained by replacing the missing element (the evolution of Q) with an *ad hoc* prescription: namely, choosing $\dot{Q}$ to enforce inclination $\iota = \mathrm{const}$. This kludge, suggested by Cutler, was shown to describe well most of the circular, inclined CB inspirals [23], and arguably it produces plausible qualitative results in the generic case [21]. Once the orbits are in hand, the inspiral gravitational waveforms can be computed using the quadrupole-moment formula and higher-order post-Newtonian emission formulas. An even better option is the fast-motion, weak-field emission formula developed by Press [27], incorporating important relativistic features such as realistic high-frequency components (which show up only at relatively high orders in the post-Newtonian formulas) and relativistic beaming (which implies a strong dependence of the waves on the direction between the source and LISA).

An alternative approach to computing CB kludge waveforms, pursued by Cutler and Barack at the AEI, is to start from *Newtonian* eccentric orbits, and then incorporate leading-order post-Newtonian effects such as precession of periastra in the orbital plane, Lense–Thirring (spin–orbit) precession of the orbital plane, and radiation-reaction–induced evolution of the integrals of the motion. Waves are then computed using a quadrupole-moment formula. The advantage of this approach is that the orbits and the waveforms can be written as *analytic expressions*, while the geodesic formalism described above always requires numerical integration; the disadvantage is that the orbits lack the strong-gravity effects expected for CB inspirals, such as extreme perihelion precession. The resulting waveforms are therefore incorrect both quantitatively *and* qualitatively; however, because all the basic physical effects are included, even with the wrong strengths, these waveforms can be very useful for a first attack to the data-analysis problem.

B. CB inspiral waveforms: status of numerical implementation

The spectrum of theoretical methods outlined above are currently being implemented as numerical codes that are capable of providing test waveforms suitable for LISA data-analysis development. In particular:

1. A code (by Cutler and Barack) is available to compute analytic pseudo-Newtonian CB inspiral orbits and waveforms, including the effects of LISA's orbital motion. We will refer to the waveforms produced by this code as "AK" (analytic-kludged). The code was improved with higher-order post-Newtonian terms for the orbital precession, which yield essential agreement with NK waveforms (see below) in the case of moderate eccentricities.

2. Codes (by Glampedakis and Kennefick, and by Hughes) are already available to compute equatorial and circular-inclined CB orbits using the Teukolsky–Sasaki–Nakamura formalism.

3. A code (by Hughes and Creighton) is available to compute generic CB orbits using the correct geodesic kinematics, a weak-field quadrupole-moment formula for the energy and angular-momentum fluxes, and Cutler's *ad hoc* prescription for $\dot{Q}$ (Glampedakis has proposed an improvement to the original formula, useful for highly inclined orbits [28]). Waveforms can be computed by a quadrupole-moment formula, or by Press' fast-motion formula, implemented and tested by Fang and Gair, and they include the effects of LISA's polarization, but not of its orbital motion. Recently, Babak and Glampedakis [29] compared the gravitational waves computed with this code (using both the quadrupole-moment and Press' formula) with Teukolsky–Sasaki–Nakamura waveforms for equatorial and circular-inclined waveforms; they point out that the quadrupole formula is surprisingly accurate,

at least for the purposes outlined in this whitepaper; however, a modified scheme that includes the effects of background curvature is desirable for perihelia $\lesssim 5M_{\rm MBH}$ [30]. We will refer to the waveforms computed using this code as "NK" (numerical-kludged).

4. A code (by Glampedakis, Kennefick, Hughes, and others) is being developed to compute generic orbits using the Teukolsky–Sasaki–Nakamura formalism, and Cutler's *ad hoc* prescription for $\dot{Q}$. Waveforms will be obtained by Press' fast-motion formula, and other additions are planned (including the parallelization of the code, and the use of improved analytic schemes for the representation of the geodesic and for the computation of MBH perturbations). We will refer to the waveforms computed using this code as "NK-2".

5. Finally, one or more codes will be eventually developed to compute generic orbits using the Teukolsky–Sasaki–Nakamura formalism, including the correct evolution of Q. These are the waveforms that will be used for actual LISA data analysis.

C. CB inspiral waveforms: template counting

The inspiral signals depend on 14 parameters: MBH mass M, MBH spin $\mathbf{S}$ (times three), CB mass μ, initial time t_0 (at a fiducial gravity-wave frequency), initial orbital plane $\hat{\mathbf{L}}_0$, initial orbital phase and line of nodes $\{\phi_0, \gamma_0\}$, distance to the source D, and position of the source in the sky $\{\phi_S, \theta_S\}$; other equivalent parametrizations are possible. We shall denote the parameters collectively as η_i. For a signal of duration $N_{\rm cycles}$, we characterize the number of templates needed to model the signal accurately[6] as

$$N_{\rm templates} \simeq (N_{\rm cycles})^{\Sigma_i p_i}\,, \tag{3}$$

where each p_i is (in our terminology) the *effective dimension* of the parameter η_i. The p_i's are in fact functions of $N_{\rm cycles}$, but a crude upper limit for very large $N_{\rm cycles}$ is one. In the case of the CB inspirals, however, we know that some parameters have null effective dimension: for instance, $p_D = 0$, because the distance to the source affects the strength, but not the shape of the signal.

Optimal (*coherent*) signal processing for a given class of signals is possible for a maximum number of cycles dictated by the available computational power. In particular, the computing power needed to process $N_{\rm templates}$ templates in real time (assuming a Nyquist frequency of 1 Hz, which is appropriate for LISA) is about $N_{\rm templates}$ flops.[7] Taking one teraflop as a reasonable computational power to devote to one seach, we get the maximum signal length that still allows optimal signal processing from

$$(N_{\rm cycles,opt})^{\Sigma_i p_i} = 10^{12}. \tag{4}$$

For a typical CB inspiral, we expect roughly 10^5 cycles over one year, with perhaps 7–10 parameters having a strong effect on the signal. Correspondingly, for $\sum_i p_i \sim 7$, we get $N_{\rm cycles,opt} \sim 50$; for such a low number of cycles, however, $\sum_i p_i$ is probably smaller, say 5, so we correct our estimate to $N_{\rm cycles,opt} \sim 250$, or to a fraction of a year.

To use the whole duration of the mission to the best possible advantage, we can use a *stacked search* [31], where the S/N for each stretch of 250 cycles is added incoherently.[8] For a signal of the same strength, the effective S/N is then a factor of $(N_{\rm stretches})^{1/4}$ lower than the optimal S/N. Stacking up stretches of 250 cycles to build the full $N_{\rm cycle} = 10^5$ signal, we have $N_{\rm stretches} \sim 400$. The equivalent S/N threshold for detection with the stacked search can then be reconstructed from

$$P_{\rm false} = N_{\rm templates} \exp\left\{-\frac{[({\rm S/N})^*_{\rm stack}]^2}{2}\right\} = \left(\frac{N_{\rm cycles}}{N_{\rm stretches}}\right)^{\Sigma_i p_i} \exp\left\{-\frac{[({\rm S/N})^*_{\rm opt}]^2}{2\sqrt{N_{\rm stretches}}}\right\}, \tag{5}$$

[6] We define operatively "modeling the signal accurately" to mean "recovering a very high fraction (perhaps 98%) of the available signal power." Cutler and Barack have pointed out that, in the context of a *stacked search* (see below), it makes sense to require that a high fraction of power be recovered *on the average*, rather than in each stack. The template counts reported in this paper are based on this definition.

[7] This is because the comparison of a signal of duration $N_{\rm cycles}P$ (where P is the period) against one template takes $\sim N_{\rm cycles}P$/sec floating point operations, using a Nyquist frequency of 1 Hz. For real-time data analysis, $N_{\rm templates}$ such comparisons need to be done in a time $N_{\rm cycles}P$.

[8] Because signal power is accumulated linearly with time, the total signal power is the same as in a coherent search. However, because noise power is normalized to a single stretch, and because noise accumulates essentially as a random walk, the total noise power is higher by a factor of $(N_{\rm stretches})^{1/2}$. The stacked-search *amplitude* S/N is then lower by a factor of $(N_{\rm stretches})^{1/4}$.

and therefore

$$(\mathrm{S/N})^*_{\mathrm{opt}} = (N_{\mathrm{stretches}})^{1/4} \sqrt{2\left[\sum_i p_i \left(\log N_{\mathrm{cycles}} - \log N_{\mathrm{stretches}}\right) - \log P_{\mathrm{false}}\right]}. \tag{6}$$

So the equivalent optimal-search threshold (which is a measure of the strength that a signal needs to have in order to be detected confidently) increases by the factor $(N_{\mathrm{stretches}})^{1/4}$ [however, it decreases slightly because of the $-\log N_{\mathrm{stretches}}$ factor under the square root,[9] which represents the decrease in the number of templates, and therefore independent statistical trials applied to the detector output].

Given a family of templates (as functions of the 14 CB-inspiral parameters), we can improve on Eq. (3) by evaluating the size of the *discrete* template bank that would approximate closely an arbitrary template chosen with the *continuous* template family. This evaluation is carried out using the *mismatch-metric* formalism [20, 32], which involves approximating the correlation between nearby templates in the parameter manifold as a Riemannian metric; $N_{\mathrm{templates}}(N_{\mathrm{cycles}})$ is then related to the *proper volume* of the parameter manifold (within the region corresponding to physical values of the parameters) with respect to the mismatch metric. As the kludge template families progress, they are being used to evaluate $N_{\mathrm{templates}}$ in this way, providing the following results:

1. *AK templates.* Because these waveforms are available in analytic form, the components of the mismatch metric can be computed analytically as integrals similar to Eq. (2), where one of the signals is differentiated twice with respect to template parameters. The numerical evaluations of the integrals, previously very slow, is now much faster thanks to the reimplementation of the expressions in Fortran.

   Cutler and Barack [33] have given a rough outline of how a stacked search could be run for CB-inspiral signals. They subdivide the template parameters in three classes: constant parameters, such as CB and MBH mass; evolving parameters, such as orbital frequency and eccentricity; phases, of which there are exactly three.[10] The basic idea is that, at the level of parameter granularity allowed by computational and probabilistic contraints, the templates are a good match for the physical signals, except that, several times per year, the three phases fall out of sync and have to be reset. In the proposed strategy, we would compute for each stack the correlation between the detector output and a bank of templates laid along *all* the parameters; we would then combine the signal power found in the stacks into parameter bins obtained by throwing away the phases, by taking equal values of the constant parameters, and by matching the evolving parameters through time using the equations of motion.

   Cutler and Barack set the available computational power at 30 teraflops, which allows the real-time processing of approximately 10^{13} templates. With these many templates, on the basis of the metrics evaluated at a few parameter-manifold points (including spin), they estimate that *coherent data analysis will be possible up to integration times of about a week.* Using the first part of Eq. (5), for $N_{\mathrm{stretches}} = 52$, $N_{\mathrm{templates}} = 10^{13}$, and $P_{\mathrm{false}} = 10^{-3}$, I find $(S/N)_{\mathrm{opt}} \sim 23$, corresponding to setting $\sum_i p_i \simeq 4$.

   Vallisneri [34] has suggested a method to reduce the number of parameters that need to be considered explicitly in computing the metric, by incorporating five angles, which describe LISA's position and orientation with respect to the inspiraling binary, into the detection statistic: namely, a detection threshold is set on the signal-template correlation *already maximized over the five angles.* The maximization can be carried out automatically (i.e., algebraically) after computing the correlation between the detector output and five subtemplates that express the symmetric, trace-free components of the (dot-dot) mass quadrupole moment of the source. This technique was investigated and tested by Pan, Buonanno, Chen, and Vallisneri to build LIGO templates for precessing NS–BH binaries [35]. Cutler and Barack [36] have estimated a reduction of ~ 200 in the number of templates obtained with this procedure, for integration times of a few days. A reasonable estimate of the computational cost of the maximized detection statistic is ten times the cost for the single-template statistic; the overall gain is therefore a factor of 20. Extrapolating crudely from Cutler and Barack's graphs [33], I estimate that coherent data analysis should then be possible for integration times of about two weeks. Using Eq. (5), for $N_{\mathrm{stretches}} = 26$, $N_{\mathrm{templates}} = 10^{13}$, and $P_{\mathrm{false}} = 10^{-3}$, I find $(S/N)_{\mathrm{opt}} \sim 19$.

   Prospects: Cutler and Barack warn that the results outlined above depend very strongly on the orbital frequency at the center of each stack; a more careful estimate would allow for stacks of different lengths, so that they

[9] That factor might disappear if the *threading* of the stacks (i.e., the identification of the parameter set that represents the same physical system in each of the binary) suffers from large uncertainties.

[10] One orbital phase, one precessional phase, and one angle that determines the direction of the periastron.

would all span approximately the same number of orbital cycles. Furthermore, the mismatch metric must be evaluated at a larger number of parameter-manifold points. However, Cutler and Barack are now in the process of writing up their research, so firmer results should be forthcoming.

2. *NK templates.* The evaluation of the mismatch metric is much more difficult for templates that are available only as numerical time series; here the metric must be reconstructed by fitting a *many-variable* quadratic form to the correlations computed numerically for a variety of parameters around a fiducial point. The difficulty is compounded by the problem that the principal axes of the metric are not generally aligned with the coordinate directions of the parameters; Gair has reported some success with an iterative scheme where a fitted approximation to the metric is used to realign the axes before gathering points for the next approximation (as first experimented in [20]), and he is about to embark in large-scale Monte Carlo evaluations. Comparisons between NK and AK metrics are hampered by the different structures of the kludged waveforms, which makes it hard to define *equivalent* sets of parameters: for instance, a pair of NK and AK templates can share the same eccentricity and periapsis and have different orbital frequencies, or they can share the same eccentricity and frequency, but have different periapses.

   Prospects: Gair, Creighton, Vallisneri, and Pan plan to start evaluating mismatch metrics on the maximized detection statistic described above. The maximization scheme relies on a specific decomposition of interferometer response that is only appropriate if gravitational waves can be described by the quadrupole-moment formula [34, 35], and if the effect of LISA's motion during each stack can be neglected. In fact, the comparison of Teukolsky-based and kludged waveforms seems to support the former hypothesis [29], while the AK estimate of allowable stack lengths seems to support the latter. Vallisneri and Pan are planning an alternative evaluation of mismatch metrics as a Fisher matrix, based on the analytic differentiation of the signal along the maximized-parameter direction [35], which should provide an important check against the Gair results. In August 2003, when Cutler visits Caltech, the AK and NK teams will cross-check conventions and results.

3. *Schutz and Sathyaprakash.* In a recent paper [38], these authors compute the mismatch metric for post-Newtonian waveforms at a few parameter points; their waveforms are comparable to the AK waveforms, and they include spin-induced precessional effects treated as in Ref. [39]. They argue that the (relative) smallness of several metric eigenvalues implies, in our language, that $\sum_i p_i \simeq 3$ (rather than 12). Schutz and Sathyaprakash advocate a multi-stage hierarchical search where one-year-long candidate signals are first determined by a non-specific time-domain filter, and then screened by matched-filtering searches on short segments; the resulting best-fit parameters are then refined on the entire year; such a search would economize on the number of effective independent statistical trials.

V. EXECUTIVE SUMMARY

Detection rates with current sensitivity. Based on Cutler and Barack's latest results (Sec. IV C 1), and on Finn and Thorne's S/N estimates [5], *a single LISA interferometric combination should detect typical* $(10 + 10^6)M_\odot$ *CB–MBH inspirals out to* ~ 2.6 Gpc *for nonspinning MBHs, and to* ~ 9 Gpc *for quasimaximally spinning MBHs* [respectively ~ 3.2 and ~ 10 Gpc using the angle-maximized statistic; these numbers should increase by a factor $\sim \sqrt{2}$ if two LISA interferometric combinations can be used together]. Extrapolating crudely from Phinney's estimates (Sec. III), one would *multiply Phinney's event rate of* 1–10/*year out to one* Gpc *by a factor between* 20 *and several hundred, depending on the distribution of MBH spins.* This extrapolation assumes that each CB inspiral can be resolved independently within the background of the other inspirals, and of the galactic and extragalactic comparable-mass binaries.

Outlook. Models for CB-inspiral orbits and waveforms are being developed slowly but surely, and promising analytical advances are reported on the evolution of Carter's constant, so far elusive. Altogether, this suggests that reliable templates suitable for parameter extraction (and therefore for the hoped scientific payoffs) will be available by the time LISA is launched. Unfortunately, the number of templates necessary to detect the inspirals is still highly uncertain, and consequently so are detection thresholds and detection distances. Much as in the case of compact binaries for LIGO, the estimation of capture rates for CB inspirals is also highly uncertain. Although it is certainly possible to improve the current framework, the question will be finally decided only by the actual detection of these sources, or lack thereof.

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