PROSPECTS FOR GRAVITATIONAL-WAVE OBSERVATIONS OF NEUTRON-STAR TIDAL DISRUPTION IN NEUTRON-STAR–BLACK-HOLE BINARIES

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The equation of state of the bulk nuclear matter inside a neutron star (NS) is still poorly understood. Thorne has conjectured that we might get useful information about it by measuring the gravitational waveforms emitted by merging NS–NS binaries, or by tidally disrupting NS's in neutron-star–black-hole (NS–BH) binaries.[1] We have strong evidence[2] that NS–NS merger waves *do* carry equation-of-state information, but they are emitted at frequencies ($\sim$ 1400–2800 Hz), which are too high for measurement by LIGO-type gravity-wave interferometers. We show that the prospects for NS/BH measurements are much brighter.[3]

We employ a highly simplified, quasi-analytic model[4] that represents the inspiraling NS as an irrotational, incompressible Newtonian ellipsoid that moves on a circular equatorial geodesic around a Kerr BH, and that is tidally distorted by the Kerr Riemann tensor. The equilibrium solutions for the ellipsoids are the classic irrotational, homogeneous *Roche-Riemann ellipsoids*,[5] which can be identified by only two NS parameters; for instance, the mass m and the radius R. Of course, not all combinations of m and R are possible for a given BH mass M, Kerr parameter a, and orbital separation r. We describe the inspiral as a sequence of circular, equatorial Kerr geodesics that shrink inward until the NS plunges into the BH (at the innermost stable circular orbit), or until it begins to disrupt tidally: this happens at the orbital separation $r_{\rm td}$ at which no equilibrium ellipsoids exist anymore.

Having measured the gravity-wave frequency $f_{\rm td}$ at which tidal disruption begins, and having determined the masses M and m from the observed inspiral waveforms,[6] we can compute $r_{\rm td}$, and then the NS radius R. So the LIGO-II observations can determine a point on the NS mass–radius curve $m(R)$, which represents the NS equation of state in our simplified analysis. We find that for a wide range of realistic parameters ($R = 8$–$16\,\mathrm{km}$ with $m = 1.4\,M_\odot$, $M = 2.5$–$40M_\odot$), tidal disruption occurs before the plunge begins; furthermore, for all realistic parameters except a very narrow range ($M < 10\,M_\odot$ and $R < 10\,\mathrm{km}$), the tidal-disruption waves fall in the range of good LIGO sensitivity, $f < 1000\,\mathrm{Hz}$.

To estimate the accuracy to which LIGO-II could measure $f_{\rm td}$ (and then R), we use *Wiener optimal filtering*:[7] if we can specify the dependence of the waveforms on $f_{\rm td}$, then we can estimate the average error in the *best fit* $\hat{f}_{\rm td}$ obtained from the signal emitted by a binary at the distance d. Because so little is known about the tidal disruption and our NS models are so crude, we use the simplest waveforms in our analysis: slow-motion, quadrupolar waveforms for point particles in circular, Keplerian orbits with quadrupole-governed inspiral. Because we expect tidal disruption to be a sudden event, we employ a toy model where the inspiral waveform dies out over a frequency band $(f_{\rm td}, f_{\rm td} + \delta f)$, where δf is consistent with a time scale of 3 orbital periods for complete disruption.

We have evaluated $\Delta\hat{f}_{\rm td}$ numerically, using LIGO-II noise curves, setting $m = 1.4\,M_\odot$, and taking low-mass BHs ($M = 2.5\,M_\odot$) at 65 Mpc, and higher-mass BHs ($M = 10,\ 20,\ 40\,M_\odot$) at 140 Mpc. These distances yield about one merger/yr according to present estimates.[8] We have then evaluated the 2σ error in the NS radius R inferred from $\hat{f}_{\rm td}$. The estimates for our fiducial decay model suggest that R may be determined with a precision of $\sim 15\%$ using a 850 Hz-narrowband LIGO-II configuration,[9] and with a somewhat worse precision using a wideband LIGO-II. If the dynamics of the decay are more favorable (but still plausible) the error might be as low as $\sim$ 6–10%. On the other hand, a 500 Hz-narrowband LIGO-II would be useful only for the heavier BH's or for the larger NS's, which have lower $f_{\rm td}$. We have also repeated our evaluation using *compressible* polytropes[4] with $n = 1$: even then, and in the most conservative decay scenario, an 850 Hz-narrowband LIGO-II should be able to provide significant information about R.

Given our crude approximations, these results can only be rough indications of the prospects for learning about NS's from tidal-disruption waveforms. They do, however, suggest that observations of tidal disruption in NS–BH binaries might be possible in $\sim$ 2006–2008 with LIGO-II, and might yield useful insights into the NS equation of state. The success of this endeavor will require the development of better theoretical and numerical techniques for modeling NS tidal disruption and computing the dependence of the disruption waveforms on the NS equation of state; we strongly advocate such an effort.

Acknowledgments

The author thanks Kip Thorne for guidance and support. This research was supported in part by NSF Grants AST-9731698 and PHY-9900776.

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