UNIVERSITÀ DEGLI STUDI DI PARMA
FACOLTÀ DI SCIENZE MATEMATICHE, FISICHE E NATURALI

DOTTORATO DI RICERCA IN FISICA
XIII CICLO, 1997/2000

Relativity and Acceleration

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Prefazione

Questa tesi contiene il lavoro che ho svolto sotto la guida del mio *tutor*, Prof. Massimo Pauri, nei tre anni (1997–2000) del corso di dottorato in fisica presso l'Università di Parma. La nostra indagine è iniziata sugli argomenti della mia tesi di laurea (l'elettrodinamica dei sistemi accelerati, la radiazione di Unruh–Hawking, e la termodinamica dei buchi neri) ma il nostro interesse si è gradualmente spostato sui fondamenti della relatività speciale e generale, e in particolare sulla fisica degli osservatori accelerati.

È difficile raccontare la meraviglia e il rapimento che ho provato nell'addentrarmi in queste regioni affascinanti della conoscenza, dove la fisica incontra la filosofia. Spero che il lettore potrà condividere i miei sentimenti almeno in parte: non certo attraverso le mie pagine, ma grazie agli autori notevolissimi che questa ricerca mi ha fatto conoscere e che sono citati nella bibliografia. Buona lettura.

Ringraziamenti. Vorrei ringraziare tutte le persone che hanno contribuito a rendere produttivi e piacevolissimi questi tre anni di dottorato, e la stesura di questa tesi. Prima di tutti Massimo Pauri, per l'amicizia e l'entusiasmo, per l'acutezza e la profondità che ha dispensato con abbondanza su queste pagine, e per la generosità con cui, tra mille impegni, ha sempre saputo trovare tempo per me. Vorrei poi ringraziare il coordinatore del corso di dottorato, Roberto Coïsson, per la sua disponibilità e flessibilità nell'aiutarmi a progettare il mio iter di dottorato un po' internazionale. Liliana Superchi ed Enrico Onofri sono stati in questi anni un punto di riferimento insostituibile: il loro aiuto e il loro sostegno sono stati impagabili. Ringrazio Luca Lusanna e Giuseppe Mambriani per l'amicizia e per le interessantissime discussioni; per i preziosi consigli sulla mia esperienza americana, Antonio Scotti e Giovanni Cicuta. Ringrazio poi il Direttore di Dipartimento, Marco Fontana, i bibliotecari Marina Gorreri, Massimo Savino e Lucia Lucchini, e gli informatici Roberto Alfieri, Roberto Covati e Raffaele Cicchese.

Ho molte ragioni per essere grato ai colleghi del Gruppo di Relatività del California Institute of Technology, Pasadena: ma in particolare, per aver letto e commentato parti di questo manoscritto, grazie a Kip Thorne, Lee Lindblom, Alessandra Buonanno e Kashif Alvi; a Kip va inoltre il merito di aver reso un po' meno ampolloso e un po' più chiaro lo stile del mio inglese. Grazie anche ai miei amici (e *fratelli*, nella discendenza scientifica da Massimo Pauri) Filippo Vernizzi, Federico Piazza e Marcello Ortaggio. Infine, grazie alla mia famiglia per l'affetto e per il sostegno (da vicino e da lontano), e grazie alla mia dolce Elisa. Ancora una volta, senza il suo amore, il suo aiuto e la sua pazienza questo lavoro non sarebbe stato possibile.

Pasadena, 25 novembre 2000

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Chapter 1

Introduction

Einstein's theory of special relativity is deeply connected with the notion of *inertial reference frame*: it is correct to say that special-relativistic theories, as described in Lorentz coordinates, *speak the language of uniformly moving observers.* Indeed, the integration between the basic postulates of the theory (the principle of relativity and the light principle), its physical substructure (Minkowski spacetime), and its basic descriptive elements (Lorentz coordinate systems) is so tight that it took many years, after the theory's inception, to unravel properly the factual from the merely descriptive (for instance, to distinguish *symmetry*, which is a consequence of the structure of spacetime, from *covariance*, which is an algebraic property of the transformations between coordinate systems).

Nevertheless, there is a special interest in the consideration of *accelerated observers*, even in a special-relativistic context. First, accelerated frames are historically the germ from which general relativity was born, both because Einstein came to the principle of equivalence through the investigation of uniformly accelerated frames, and because Einstein's primary pretense for the general theory was to extend the *relativity* of physical laws from inertial to generic observers. Second, there are special topics in relativistic theories (such as the now-famous Unruh effect, or the problem of radiation reaction) where it appears that physical insight can benefit much from a subjective description *made from the point of view of accelerated observers.* For instance, the Unruh effect (according to which an accelerated detector interacts with the ground state of a quantum field as if it was in contact with a thermal bath of particles) can be understood in terms of field quantization in accelerated coordinates; and the paradox of the radiation of falling charges (according to which a charge in a homogeneous gravitational field should fall differently from an uncharged test body, contrary to the principle of equivalence) can be elucidated by rewriting Maxwell's equations in the accelerated frame.

In this work, we study the *physics of accelerated observers* from several points of view. In Chapter 2, we identify the origin of the Unruh effect (and of its analog in black-hole spacetimes, the Hawking effect) in the *classical* principle of *perspectival semantics*, according to which some familiar notions defined in special-relativistic theories (such as *particle* and *radiation*) inevitably lose their coherence when they are transported to accelerated frames or to curved spacetimes. In Chapter 3, we propose a general scheme to build an accelerated system of coordinates (*Märzke-Wheeler coordinates*) adapted to the motion of a generic accelerated observer, and we suggest two applications for this new system. It turns out that the definition of coordinate systems (both inertial and accelerated) is intimately tied to the choice of a relation of *distant simultaneity* between events: in Chapter 4, we review the perennial debate (among relativists and philosophers of physics alike) on the *conventionality of simultaneity* in special relativity, and we examine the conventionality of *Märzke-Wheeler simultaneity.* More in detail, here is the synopsis of the three chapters of this thesis.

Chapter 1: Classical roots of the Unruh and Hawking effects

Although the Unruh and Hawking effects are commonly considered as pure quantum mechanical phenomena, we argue that they are deeply rooted at the classical level. We believe that we can get very useful insights on these effects if we consider how the special-relativistic notion of particle becomes *blurred* when it is employed in general-relativistic theories, or in special relativity, but for accelerated observers. This *blurring* is an instance of a more general behavior (*perspectival semantics*) that arises when the principle of equivalence is used to generalize special-relativistic theories (be they quantum or classical) to curved spacetimes or accelerated observers. We support our claim by analyzing a classical analogue of the Unruh effect that stems from the noninvariance of the notion of electromagnetic radiation, as seen by inertial and accelerated observers. We use four *gedanken-experimente* to illustrate this example, and we review the related debate on the radiation of uniformly falling charges.

Chapter 2: Märzke-Wheeler coordinates for accelerated observers in special relativity

In special relativity, the definition of coordinate systems adapted to generic accelerated observers is a long-standing problem, which has found unequivocal solutions only for the simplest motions. We show that the *Märzke-Wheeler construction*, an extension of the Einstein synchronization convention, produces accelerated systems of coordinates with desirable properties: (a) they reduce to Lorentz coordinates in a neighborhood of the observers' worldlines; (b) they index *continuously* and *completely* the *causal envelope* of the worldline (that is, the intersection of its causal past and its causal future: for well-behaved worldlines, the entire spacetime); (c) they provide a smooth and consistent foliation of the causal envelope into spacelike surfaces. We compare *Märzke-Wheeler coordinates* with other definitions of accelerated coordinates, we examine them in the special case of *stationary motions*, and we employ the notion of *Märzke-Wheeler simultaneity* to clarify the relativistic *paradox of the twins*. Finally, we suggest that field quantization in Märzke-Wheeler coordinates could solve a well-known inconsistency in the theory of the Unruh effect (namely, the circumstance that quantization in naive rotating coordinates is inconsistent with the measurements of a rotating detector).

Chapter 3: The Conventionality of Simultaneity

The problem of the *conventionality of simultaneity* in special relativity has been the subject of a vigorous discussion in the last 30 years: the issue is whether the *Einstein synchronization convention* (which defines Lorentz inertial time) is a fundamental constituent of special relativity or whether other conventions can still preserve the physical content of the theory. We review the main contributions to this debate, and we find that the evidence for the *nonconventionality* of Einstein synchronization appears very compelling. We extend the discussion to accelerated observers in special relativity, and we make the case for the nonconventionality of *Märkze-Wheeler* simultaneity.

Chapter 2

Classical Roots of the Unruh and Hawking Effects

2.1 Introduction

For the last three decades, the Unruh and Hawking effects have been deservedly among the most widely discussed and popularized subjects in theoretical physics. A strong part of their folklore is the conviction that these effects have an eminently *quantum mechanical* character. For instance, many authors write that black holes are truly *black* by classical physics, so the analogy between black-hole mechanics and thermodynamics would not be complete without the inclusion of quantum mechanics, which provides the thermal emission of particles from black-hole horizons [*Hawking effect* (Hawking, 1975)]. And again, the fact that the Minkowski vacuum contains particles that can be seen by an accelerated detector [*Unruh effect* (Unruh, 1976)] is perceived as a modern quantum marvel *on a par*, say, with quantum tunneling and EPR effects.[2] In this thesis we claim instead that both the Unruh and the Hawking effect have a clear classical counterpart, and that they can be understood as typical examples of the *perspectival semantics* that arises in the context of the difficult migration from special relativity to curved-spacetime physics, or simply to an extension of special relativity which includes accelerated observers (Vallisneri, 1997).

The assertion that the Poincaré group is the global symmetry group of spacetime has been seminal to the great theoretical synthesis of the first half of this century, begun with the full recognition of Maxwell's electromagnetism as a special-relativistic theory, and beautifully climaxed with quantum field theory. So the concepts and the interpretive paradigms of

[1] Originally published as M. Pauri and M. Vallisneri, *Found. of Phys.* **29**, 1499–1520 (1999). gr-qc/9903052.

[2] As insightfully discussed by Sciama (1979), these phenomena bring together Einstein's independent legacies, fluctuation theory and relativity, in a very intriguing way.

electromagnetism and quantum field theory refer naturally to the privileged class of the *inertial observers* of special relativity, who are closely associated with the symmetry properties of the theory (see Sec. 4.3.1). Now, the equivalence principle of general relativity still ensures that the Lorentz group is the symmetry group of spacetime, but only locally: this local character becomes crucial when we try to generalize to curved-spacetime geometries the special-relativistic concepts and paradigms that are based on the *global* symmetry of Minkowski spacetime.

In Sec. 2.2 we shall argue that the essence of the Unruh and Hawking effects can be understood from this point of view, even before considering their quantum character: we shall see that the special-relativistic notion of *quantum particle* becomes *slippery* when we try to extend it to curved spacetimes or to noninertial observers, and that this slipperiness is the source of the Unruh and Hawking effects.

In Sec. 2.3 we shall see that the *same* ambiguity befalls the *entirely classical* concept of *electromagnetic radiation*, and we shall examine the especially instructive paradox of a charge falling in a constant homogeneous gravitational field. Namely, because it emits radiation, a falling charge might be distinguished from a falling, uncharged body, suggesting a violation of the equivalence principle of general relativity. We shall deliberately introduce the issue in a blurred way that echoes its initial appreciation in the literature as a borderline case between special and general relativity; this presentation makes the paradox most apparent. But the paradox fades if we place the question fully in the theoretical framework of general relativity. The solution is that the notion of electromagnetic radiation *is not invariant* with respect to transformations between inertial and accelerated frames, so radiation can be produced or transformed away by changing the state of motion of the observer.

We regard this illusion as a veritable forerunner of the Unruh and Hawking effects, and we submit that these effects are, in R. Peierls' definition (1979), *intellectual surprises* that could have been foreseen much earlier; the reason they were not lies in the difficult epistemic *upgrade* required to switch from the special to the general-relativistic worldview.

2.2 Classical nature of the Unruh and Hawking effects

As many authors have underlined, the most transparent explanation of the Unruh and Hawking effects is that they are brought about by the presence of conflicting definitions for the notion of *quantum particle*. The essential ingredients in the standard quantization of free field theories are the normal-mode decomposition of field operators, and the distinction between *positive-frequency* and *negative-frequency modes*, which fixes the identity of particles, of antiparticles, and (most important) of the vacuum state. However, there are infinitely many ways to accomplish this decomposition,[3] which correspond roughly to all the possible choices of a complete set of solutions for the classical wave equation. When we build a correspondence between quantum field theories based on different decompositions,[4] we find some cases where a vacuum state is mapped to a *particulate* state.

In special-relativistic theories, there is an obvious criterion to select one particular quantization: we pick the classical solutions of definite frequency with respect to Lorentz coordinate time. In doing so, we ensure a *covariant* notion of particle that is adequate for all inertial observers. However, if we extend our scope beyond inertial observers, we find that in some situations there can be more than one logical choice of modes.

A first example is the Unruh effect (Unruh, 1976), which takes an observer traveling through Minkowski spacetime along a uniformly accelerated worldline. The observer naturally uses modes of definite frequency with respect to her *proper time*: she then finds that the Minkowski vacuum (the vacuum state, according to the mode decomposition based on Lorentz coordinate time) corresponds to a thermal bath of particles according to an accelerated-mode decomposition.

This result is considered *robust*, because it can be derived by an altogether different approach (Unruh, 1976). By standard approximation theory, we find that a quantum detector moving along a predetermined, accelerated worldline will thermalize on interaction with the Minkowski vacuum. It is a well known result that the energy-absorption rate of the detector is determined essentially by the Fourier transform of the *field autocorrelation function* (taken along the detector's worldline, and with respect to the detector's proper time):

$$R(\omega) = \int_{-\infty}^{+\infty} \mathrm{d}\tau \, e^{-i\omega\tau} \langle 0|\hat{\phi}(x^\mu(0))\hat{\phi}(x^\mu(\tau))|0\rangle. \tag{2.1}$$

The Wiener–Khinchin theorem states that the Fourier transform of a signal's autocorrelation is just the signal's power spectrum. The response of the detector, therefore, is correlated to the energy content of the field (in a specific manner that depends on the form of the energy–momentum tensor of the field, and on the interaction Hamiltonian that couples the field and the detector). Therefore, when the detector reports a thermal signal, we can interpret the signal as proving the presence of a thermal bath of particles. Any energy absorbed by the detector, however, must come ultimately from the classical agency that keeps the detector on its worldline.

[3]See for instance (Wald, 1994).

[4]Even if different choices of the modes lead in general to *unitarily inequivalent* theories (Wald, 1994), it is always possible to establish an arbitrarily accurate correspondence between the states of any two theories, using the so called *algebraic approach* (Haag and Kastler, 1964).

Moving on to curved spacetimes, consider the Hawking effect (Hawking, 1975), which takes a quantum field living on a black-hole background geometry (more precisely, on the geometry of a spherically symmetric distribution of matter that collapses to a black hole). The symmetries of this spacetime hint to two natural definitions of quantum particle: one of them is appropriate for the observers who inhabit the early stages of collapse, when spacetime is still approximately Minkowskian; the other one is appropriate for the late observers who witness the final phase with a stationary black hole. The vacuum state, as it is defined by early observers, appears to the late observers as a thermal bath of particles coming from the direction of the black hole. Again, this conclusion can be strengthened by the consideration of quantum detectors traveling through Schwarzschild spacetime (see for instance Vallisneri, 1997).

2.2.1 Slippery notions and interpretive illusions

A *physical theory* consists loosely of three interpenetrating bodies of knowledge: an *axiomatic structure*, which identifies the principles and the laws of the theory, and the consequences that can be deduced from them; an *operative interface* to experimentation, which is often coupled with a set of defining or *encyclopaedic* experimental results; and a *semantic framework* of interpretations and paradigms, which are necessary to ascribe meaning to the observed physical world. Take for instance standard one-particle quantum theory: the axiomatic structure could be the one explained in Dirac's *Quantum Mechanics* (1958), whereas the interpretive framework could be the *Copenhagen interpretation*.

Each of these three elements evolves differently with time. Predictably, the semantic framework is the fluidest element: often it depends on unspoken perceptions and understandings, and only rarely it resides organically in written documents. The evolution of the semantic framework can be traced through the history of its blocks, *physical notions*: some notions are doomed to extinction (think of the ether); others pass unscathed or even augmented from a successful theory to the next one (think of mass and energy); and others again are subject to curious blurrings and crossbreedings (think of particles and waves after the quantum revolution). This *memetics* of notions (see Dawkins, 1976) is indeed one of the most charming and enjoyable aspects in the history of theoretical physics.

We will say that we are in the presence of *perspectival semantics* when, within a theory, we can assign distinct *semantic contents* to the same *physical information*, according to different but equally legitimate readings. This happens for special-relativistic quantum field theory when it is extended to curved spacetimes, or to accelerated observers. Even if Einstein provided the principle of equivalence to ferry across special-relativistic physics to general-relativistic shores, the old semantics cannot always cope with this upgrade:

some notions get *slippery*, or become afflicted by paradoxes.

The Unruh and Hawking effects are *instantiations of perspectival semantics where the ambivalent information is the value of the field*; the perspectival interpretation of the field points to the failure of the notion of particle. Let us then dissect this very notion. We feel entitled to speak of *quantum particles* when we remark a certain *periodic* structure in the temporal and spatial dependence of the field. This attribution of meaning is not new to quantum field theory, but it can be traced etymologically to certain basic notions of quantum field theory's *parent theories*:

1. in *one-particle quantum mechanics*, we think of the solutions to the wave equation as describing a particle;

2. in *classical nonrelativistic mechanics*, position and momentum are *fundamental observables*, which define fully the location of the representative point in phase space;

3. in *relativistic classical mechanics*, the fundamental status of the position observable is somehow weakened (due to problems of covariance); on the other hand, the energy–momentum four-vector gains clout as the *essential attribute* of a relativistic particle;

4. again *in relativistic classical mechanics*, energy and momentum are the generators of infinitesimal translations in time and space; so Fourier modes are identified as waves (and therefore, particles) of definite energy and momentum.

Because of the covariance properties of the energy–momentum four-vector, all the inertial observers in Minkowski spacetime perform *compatible* frequency analyses of the same field information, so they all come up with compatible statements about the presence of particles. When we try to enlarge quantum field theory to accommodate *accelerated observers* in Minkowski spacetime or generic observers in *curved spacetimes*, we get the Unruh and Hawking effects. Because we are *outside* the compatibility domain of the notion of particle, a legitimate frequency assessment can ascribe a particulate content to quantum states that from the inertial point of view are devoid of particles.

2.2.2 The Unruh and Hawking effects in classical field theory

We come now to our claim about the Unruh and Hawking effects. The normal-mode decomposition of the field belongs arguably to the *classical* domain: for instance, in classical field theory we can write a real scalar field as a sum of a complete set of positive-frequency, orthonormal modes,

$$\phi(x^\mu) = \sum_i a_i \psi_i(x^\mu) + a_i^* \psi_i^*(x^\mu); \qquad (2.2)$$

we can then interpret the coefficients a_i and a_i^* as expressing the presence of the single wave modes in the overall configuration of the field. Quantum field theory is obtained by promoting the coefficients a_i to Fock operators. We can then read the particle content of any quantum state by means of the *number operators*, $N_i \equiv a_i^\dagger a_i$.

Suppose we set up two *competing decompositions* for the field (just as happens in the Unruh and Hawking effects). The transformation between the coefficients (or operators) of the two decompositions *will not depend on the classical or quantum procedure used to read the field signal*; it will depend only on the way in which one set of modes can be written in terms of the other (in practice, it will depend on the scalar products between the modes from the two sets[5]). As we have already remarked, the essence of the Unruh and Hawking effects resides in this transformation, which we now come to recognize as *originally classical.* Moreover, Eq. (2.1) parallels closely the expression found by Planck (1900) for the rate at which a classical charged harmonic oscillator absorbs energy from a statistical radiation field:

$$R_{\mathrm{cl}}(\omega) = \int_{-\infty}^{+\infty} \mathrm{d}\tau\, e^{-i\omega\tau} \langle \phi(x^\mu(0))\phi(x^\mu(\tau))\rangle; \tag{2.3}$$

here ω is the natural frequency of the oscillator, and $\langle \ldots \rangle$ denotes an *ensemble* average.

If our considerations are correct, then classical field theory should exhibit the Unruh effect. Does it? Not if we take the *fundamental configuration* of classical field theory to be an everywhere null field, which is truly a *universally* invariant configuration! No matter how we read a null signal, it will always remain null. In quantum field theory, instead, the nonvanishing fluctuations of the vacuum state always provide a fundamental signal that makes the Unruh and Hawking *perspectival effects* possible.

[5]This is true already at the classical level. The usual way to define scalar products in free quantum field theories is to adapt the *symplectic structure* of the classical–wave-equation solution space (see for instance (Wald, 1994)). This procedure ensures the conservation of scalar products throughout evolution. At the classical level, these scalar products can be used to set up a conserved spectral decomposition for any solution: that is, any solution can be seen as a superposition of evolving wave modes.

If we establish a second decomposition alternative to Eq. (2.2),

$$\phi(x^\mu) = \sum_i c_i \xi_i(x^\mu) + c_i^* \xi_i^*(x^\mu),$$

we then get the new coefficients and the new creation and destruction operators as

$$c_j = \sum_i \alpha_{ij} a_i + \beta_{ij}^* a_i^*,$$
$$\alpha_{ij} = (\xi_j, \psi_i), \qquad \beta_{ij} = (\xi_j^*, \psi_i),$$

where we use the scalar product under which the ξ_i are a complete, orthonormal set.

There are two ways to introduce such a fundamental signal in classical field theory: the first is simply to bring in classical sources, and to examine the wave-mode content of the resulting inhomogenous solutions of the wave equation. Higuchi and Matsas (1993) define a *classical particle number* as energy density per unit frequency divided by frequency, and proceed to show that the relation between the particle numbers computed in inertial and accelerated coordinates is consistent with the existence of an Unruh thermal bath. A related approach is due to Srinivasan and colleagues (1997a; 1997b).

The second route is to postulate that the fundamental configuration of the classical field consists of an incoherent superposition of plane waves, which endow the vacuum with a zero-point energy of $\hbar\omega/2$ per mode; the phases of the waves are assumed to be random variables with uniform and independent distributions. Such a theory is known as *stochastic classical field theory*, and (in its specialization to electromagnetism) it was introduced by Marshall (1963; 1965); the constant $\hbar$ is imported into the classical framework by requiring the mean-square displacement of a charged harmonic oscillator to be the same as in quantum theory. The fundamental signal of stochastic classical field theory makes the Unruh effect possible (Boyer, 1984).

Our arguments for the existence of the Unruh effect in classical field theory can be reproduced for the fields that inhabit a gravitational-collapse spacetime. Therefore, there must be as well a *classical Hawking effect*. It is interesting to ask whether these classical homologues could have been noticed during the development of classical electromagnetism; then the Unruh and Hawking effects would have been derived subsequently as quantum versions of classical effects. The answer, though, is probably negative. The classical Unruh and Hawking effects have a definite retrospective flavor, in part because the notion of wave mode has a weaker semantic content then the notion of particle, and in part because in the classical domain there is no fluctuating vacuum to highlight the phenomenon.

Nevertheless, we believe that the Unruh and Hawking *perspectival effects* could have been predicted earlier, by a different route: that is, through the analogy with the slippery notion of radiation found in the extension of classical electromagnetism to general relativity. *Slippery radiation* is the subject of the next section.

2.3 The equivalence principle paradox

One of the challenges posed by the advent of general relativity to the established comprehension of the physical world was the apparent conflict between the principle of equivalence and the well established fact that accelerated charges radiate. This conflict can be spelled out by the following *Gedankenexperiment*: let us move to a laboratory setting on Earth, and

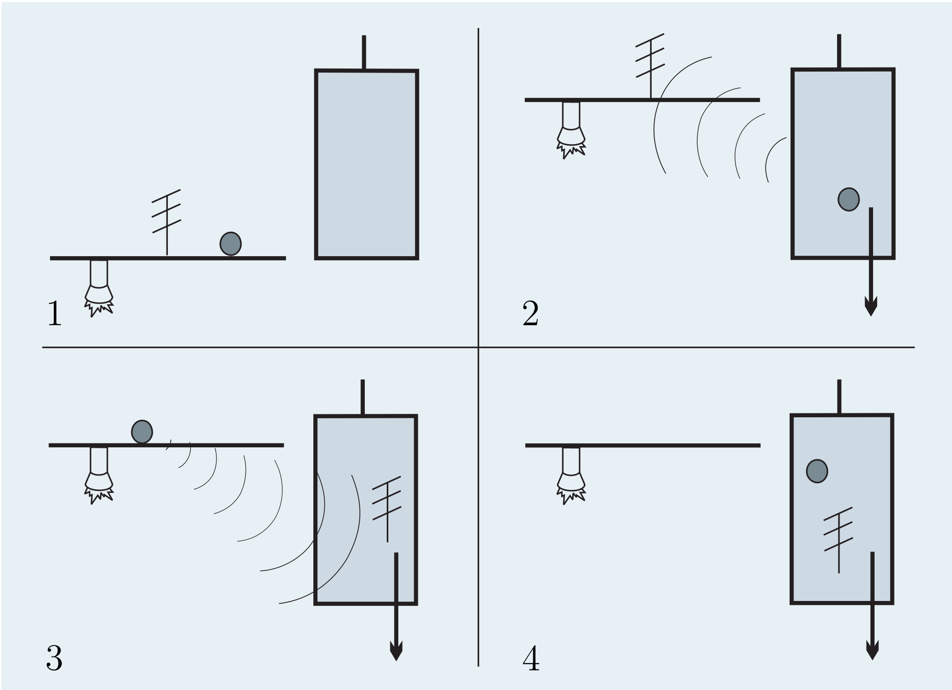

Figure 2.1: Four *Gedankenexperimente*: To the right of the laboratory frame, *supported* by a compensating agency (rockets), is our imaginary *Einstein's elevator*, which falls freely (except for experiment 1) in the gravitational field of the Earth. We assume the gravitational field to be homogeneous.

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do tests with a system that consists of a pointlike electric charge and of a detector of electromagnetic radiation. We will check whether the detector registers any radiation when the system is set up as follows (see Fig. 2.1):

1. we support both the charge and the detector in the Earth's gravitational field;

2. we support the detector, and we let the charge fall freely;

3. we let the detector fall, and we support the charge;

4. we let both the detector and the charge fall freely.

If we are willing to concede that our laboratory is small enough compared to the Earth, we can work in the idealization that the detector and the charge are immersed in a *constant homogeneous gravitational field*. Under this condition, falling objects move along uniformly accelerated trajectories (possibly relativistic ones) in the *vertical* direction.

Let us first consider experiments 1 and 2. Our prerelativistic intuition suggests that the charge at rest will emit no radiation, whereas the falling charge will; moreover, because the falling charge will lose energy to electromagnetic radiation, it will fall more slowly than a similar uncharged body. However, the *equivalence principle* of general relativity, at least in the case of homogeneous (*apparent*) fields, requires charged test particles to follow the same geodesics as uncharged ones![6]

By 1960, the very existence of radiation emitted by uniformly accelerated charges was still in dispute. In V. Ginzburg's words (1970), this is one of the "perpetual problems" of classical electrodynamics; indeed, its discussion continued for decades. M. Born's original solution (1909) for the field of a uniformly accelerated charge was interpreted divergingly as implying the emission of radiation (Schott, 1912, 1915; Milner, 1921; Drukey, 1949; Bradbury, 1962; Leibovitz and Peres, 1963; Grandy, 1970) or its absence (von Laue, 1919; Pauli, 1921; Rosen, 1962). Most notably, in his 1921 *Enzyklopädie der Matematischen Wissenschaften* article, Pauli ruled that uniformly accelerated charges do not radiate.

In Sec. 2.3.1 we briefly summarize the debate, and we see that uniformly accelerated charges *do* radiate according to the standard Larmor's formula,

$$\mathcal{R} = \frac{2}{3}\frac{e^2 a^2}{c^3}. \tag{2.4}$$

Once the presence of radiation is established, we are left with a contradiction with the equivalence principle that attracted by itself an extensive literature (Bondi and Gold, 1955; Fulton and Rohrlich, 1960; Rohrlich, 1961, 1963; Mould, 1964; Kovetz and Tauber, 1969; Ginzburg, 1970; Boulware, 1980; Piazzese and Rizzi, 1985). For instance, it has been argued (Bondi and Gold, 1955; Fulton and Rohrlich, 1960) that radiation can be measured only at a large distance from the falling charge, but that (by various considerations) we cannot postulate homogeneous gravitational fields that extend far enough to accommodate significant measurements; therefore, the problem of radiation in a homogeneous field is badly posed.

Yet we can find a more satisfactory resolution by framing the issue within our modern understanding of general relativity (Rohrlich, 1963; Kovetz and Tauber, 1969; Ginzburg, 1970). The (strong) principle of equivalence (Weinberg, 1972; Misner et al., 1973; Ciufolini and Wheeler, 1995) can be formulated as stating that *the special-relativistic equations of physics are valid, unmodified, in (local) inertial reference frames.* Coming to our experiments, under the hypothesis of a homogeneous gravitational field, Maxwell's special-relativistic equations are valid *globally* throughout spacetime, but only when

[6] Of course, because our charge is still a *test* particle, we work under the assumption that neither the mass of the charge nor the mass of the electromagnetic field are relevant for spacetime geometry.

they are written in the *freely falling reference frame.*

Therefore, the conjunction of Larmor's formula (2.4) with the principle of equivalence should not be used to predict the outcome of the supported experiments 1 and 2, but rather of experiments 3 and 4. In experiment 4, the freely falling system of detector and charge will behave exactly as a similar system at rest in the absence of gravitation, so the detector will report no radiation. In experiment 3, the supported charge will be accelerated relatively to the freely falling detector, emitting radiation as given by Eq. (2.4). This radiation is a *consequence* of the equivalence principle, rather than a violation!

Now, if the detection of radiation depended only on the state of motion of the charge, we would get a troubling result for experiments 1 and 2, where the detector is supported: contrary to our earlier intuition, the charges that are accelerated with respect to the laboratory reference frame (experiment 2) would not radiate, whereas the charges at rest in the laboratory frame (experiment 1) *would* radiate. The latter conclusion is especially embarrassing, because it is not clear how a continuous transfer of energy can be obtained in a *stationary* physical system.

The problem here is that we cannot extend the results obtained in the inertial frame to the supported experiments. That is, we cannot infer the readings of the supported detector from those of the inertial one, but we must explicitly derive them within a suitable *extension* of the special-relativistic theory of electromagnetism. There are several ways to do so: by modeling a simple radiation detector and examining its response to electromagnetic fields while the detector undergoes acceleration (Mould, 1964); by using a weak field approximation to general relativity (Kovetz and Tauber, 1969); or by evaluating the flux of the Poynting vector through spherical surfaces at rest in the supported frame (Piazzese and Rizzi, 1985).

Following Rohrlich (1963), in Sec. 2.3.2 we shall instead seek an accelerated set of coordinates that are especially adapted to observers supported in a constant homogeneous gravitational field. In this scheme, we predict the outcome of experiments 1 and 2 by taking the electromagnetic field tensors found in the inertial system for supported and freely falling charges, and by transforming the tensors to supported coordinates. In Sec. 2.3.3 we shall see that with respect to the supported coordinates, the charge at rest has a field that is very nearly of Coulomb form (experiment 1); and that the falling charge *does* emit radiation (experiment 2).

This procedure [in accord with with Mould (1964) and with Kovetz and Tauber (1969)] shows that the results intuitively expected from the supported experiments are correct to a very good approximation, at least for reasonable gravitational accelerations.[7] We find also that that *the very notion of radiation is not invariant with respect to transformations from*

[7] See Eq. (2.19).

inertial to accelerated reference frames. As noticed by Sciama (1979), *an accelerated* (in this case, supported) *observer will detect radiation where a freely falling observer sees only a pure Coulomb field:*[8] this phenomenon is very similar to the appearance of virtual particles in the Unruh effect. As in the Unruh effect, the energy that is absorbed by the accelerated observer must ultimately come from the agency that enforces the acceleration, rather than from the *putative* source of radiation (the charge). The source of the energy is discussed in Sec. 2.3.4.

2.3.1 The radiation of uniformly accelerated charges

In special relativity, we define *uniformly accelerated motion* by requiring that the worldline $x^\mu(\tau)$ have a four-acceleration $a^\mu = d^2x^\mu/d\tau^2$ of constant norm $(a^\mu a_\mu)^{1/2} = g$, or equivalently that the three-acceleration $\mathbf{a}(\tau)$ be a constant vector in the instantaneous rest frame of the worldline. If we restrict our attention to the motions that take place on a two-dimensional spacetime plane,[9] we get (up to Poincaré transformations) the worldline

$$\begin{cases} t = g^{-1}\sinh g\tau, \\ x = 0, \\ y = 0, \\ z = g^{-1}\cosh g\tau \end{cases} \tag{2.5}$$

(see Pauli, 1921; Rohrlich, 1965; Misner et al., 1973). Since these equations describe a hyperbola in the zt plane, this motion is also called *hyperbolic*, in contrast with the *parabolic* free fall of Galilean mechanics. The coordinates used to write Eq. (2.5) belong to the *instantaneous Lorentz rest frame* of the moving point, at the proper time $\tau = 0$. The trajectory is invariant with respect to Lorentz boosts along the z axis, which merely shift the trajectory along itself; indeed, the boosts amount to simple translations in proper time, which transform between the instantaneous rest frames at different proper times.

Is Eq. (2.5) also the correct worldline for a charged particle coupled to the electromagnetic field? It appears to be so, because if we insert it into the standard Dirac–Lorentz equation (Dirac, 1938; Jackson, 1962; Rohrlich, 1965),

$$ma^\mu - F^\mu_{\text{ext}} = \frac{2}{3}\frac{e^2}{c^3}\left(\dot{a}^\mu - \frac{a^\alpha a_\alpha}{c^2}u^\mu\right), \tag{2.6}$$

[8] Experiment 2 is the classical analog of the quantum Unruh effect, while experiment 4 corresponds to the quantum statement that an inertial detector sees no particles in the Minkowski vacuum. Levin and colleagues (1992) discuss what amounts to a quantum analog of experiments 1 and 3. They introduce a quantum field that lives in a uniformly accelerated cavity, and study its interactions with comoving or inertial detectors.

[9] Relaxing this hypothesis yields the larger class of *stationary trajectories*, which are discussed in Ch. 3 (Sec. 3.3) and in App .B.

the radiative damping term on the right vanishes. So the trajectory (2.5) solves Eq. (2.6) for a suitable external field F^{μ}_{ext}. This circumstance has been the root of many misgivings: because apparently the charge loses no mechanical energy to radiation, it seems natural to conclude that there is no radiation at all. We will come to this in a moment.

The electromagnetic fields associated with hyperbolic motion were first derived explicitly by Born (1909), and they can be expressed in the usual cylindrical coordinates (t, ρ, ϕ, z) as

$$\begin{cases} E_\rho = 8eg^{-2}\rho z/\xi^3, \\ E_\phi = 0, \\ E_z = -4eg^{-2}(g^{-2} + \rho^2 + t^2 - z^2)/\xi^3, \\ H_\rho = 0, \\ H_\phi = 8eg^{-2}\rho t/\xi^3, \\ H_z = 0, \end{cases} \tag{2.7}$$

where $\xi = [(g^{-2} - \rho^2 + t^2 - z^2)^2 + 4g^{-2}\rho^2]^{1/2}$ (Fulton and Rohrlich, 1960). Under the hypothesis of *retarded potentials*, the fields must be restricted to the *causal future* $z + t > 0$ of the charge. This condition was not enforced in Born's original solution, and it was introduced by Schott (1912; 1915). Bondi and Gold (1955) patched the solution further by introducing Dirac-delta fields on the surface $z + t = 0$, where otherwise the field would not satisfy Maxwell's equations.

The magnetic field and therefore Poynting's vector vanish throughout space at time $t = 0$. By symmetry, they must also vanish in every instantaneous rest frame, at all the events that in that frame are simultaneous with the charge at the origin. Pauli (1921) concludes that "there is no formation of a wave zone nor any corresponding radiation".

Now, the notion of electromagnetic radiation is usually associated with two connected physical facts:

1. the fields that originate at an event along the worldline of the charge, and which propagate outward on the lightcone, consist both of a Coulomb term decreasing as $1/R^2$ (where R is the radius of the lightcone in any given Lorentz frame) *and* of a $1/R$ *radiation term*, which eventually comes to dominate the field in the so called *wave zone*;

2. the *radiation term* arises because, at successive instants, the accelerating charge is not in the right position to support its previous Coulomb field. Thus a portion of field is effectively *splintered away*: it takes on an independent existence and travels outward from the charge, at the speed of light, carrying its own endowment of energy–momentum.

Following Fulton and Rohrlich (1960), we adopt a *local, Lorentz-invariant criterion* to decide if a charge is *instantaneously* radiating at the event $x^{\mu}(\tau)$

along its worldline: we evaluate the flux of the energy-momentum tensor $T^{\mu\nu}$ through light spheres centered at $x^\mu(\tau)$. In the limit of infinite radius for the sphere,[10] we get a unique four-vector that can be written from the kinematic parameters of the charge's trajectory:

$$\frac{dP^\mu}{d\tau} = \frac{2}{3}\frac{e^2}{c^3}(a^\alpha a_\alpha)u^\mu. \tag{2.8}$$

According to this criterion, the uniformly accelerated charge radiates with exactly Larmor's power, $\mathcal{R} = 2e^2g^2/3c^2$. As for Pauli's objection, it does not matter if Poynting's three-vector vanishes on a constant-time surface in each instantaneous reference frame, because the transfer of energy–momentum must be evaluated using the fully relativistic tensor $T^{\mu\nu}$. We would get $\mathcal{R} = 0$ only if Poynting's vector, as expressed in the instantaneous rest frame, were null on the lightcone centered on the charge, rather than on the spacelike surface $t = 0$.

Finally, we are left to prove the conservation of energy. Indeed, if the external force F^μ_{ext} of Eq. (2.6) is entirely transformed into kinetic energy, from where does the radiated energy come? We may answer this question by realizing that hyperbolic motion describes the hardly physical situation of a charge incoming from $z \to -\infty$ and leaving for $z \to \infty$, with asymptotic speeds that approach c. So when we ask about the conservation of energy, we are in fact trying to balance infinite quantities: we should expect to do this, in some sense, *in the limit*. Consider instead a trajectory built by attaching two portions of uniform motion to a finite segment of uniformly accelerated motion. At the junctions, the acceleration must necessarily be nonuniform; it is just there that radiation reaction acts to ensure that the total work exerted by F^μ_{ext} is equal to the increase in the kinetic energy of the charge, *plus* the energy radiated to infinity (Ginzburg, 1979; Tagliavini, 1991). We can imagine that the energy flux radiated by the charge during the uniformly accelerated motion is being borrowed from the *divergent* energy of the electromagnetic field near the charge, which effectively acts as an infinite *reservoir*. While draining energy, the field becomes more and more different from the pure velocity field of an inertial charge; when hyperbolic motion finally ends, the external force must provide all the energy that is necessary to reestablish the original structure of the field.[11]

2.3.2 Construction of a supported frame in a constant homogeneous gravitational field

Following Rohrlich (1963; 1965), we now seek a set of noninertial coordinates to describe the physics seen by supported observers. These observers follow a

[10] If the limit is to be finite, the field must have a $1/R$ asymptotic behavior, because $T^{\mu\nu}$ is quadratic in $F^{\mu\nu}$.

[11] D. Tagliavini, personal communication (March 1997).

nongeodesic trajectory through Minkowski spacetime, and from their point of view, cronogeometry appears to be static (that is, the speed of clocks and the length of objects does not vary with time) and flat (spacetime is Minkowskian); moreover, *spatial* geometry appears to be *homogeneous* in the two horizontal directions. We shall then define a *constant homogeneous gravitational field* as a *flat static metric that is manifestly invariant under translations and rotations in a spatial plane.* We further impose the requirement that, in these coordinates, the geodesic equation reproduce the correct Newtonian behavior in the nonrelativistic limit.

The most general metric that satisfies these properties can be written as[12]

$$ds^2 = -D(z')\,dt'^2 + dx'^2 + dy'^2 + (\sqrt{D(z')}'/g)^2\,dz'^2, \tag{2.9}$$

where, because of the Newtonian limit, $D(z')$ is required to approximate $1+2gz'$ to first order in gz'. The supported observers inhabit the worldlines of constant x', y' and z'. There are several explicit possibilities for $D(z')$, because we have the freedom to synchronize clocks differently at different heights. For instance, one of the possible metrics is

$$ds^2 = -(1+2gz')\,dt'^2 + dx'^2 + dy'^2 + (1+2gz')^{-1}dz'^2, \tag{2.10}$$

where $D(z')$ coincides with its nonrelativistic limit. The most useful choice for $D(z')$, however, yields *Rindler's metric*

$$ds^2 = -(1+gz')^2dt'^2 + dx'^2 + dy'^2 + dz'^2, \tag{2.11}$$

which implies a linear variation of clock speed by height. Einstein used this metric implicitly in his seminal argument about gravitational energy and the speed of clocks (Einstein, 1907). Note that, independently of $D(z')$, it is always possible to put the metric in the Rindler form (2.11) by introducing the new vertical coordinate z'' defined by $1 + gz'' = \sqrt{D(z')}$.

The transformation equations between inertial coordinates and the accelerated coordinates that lead to Eq. (2.9) are given (up to Poincaré transformations) by

$$\begin{cases} t = g^{-1}\sqrt{D(z')}\sinh gt', \\ x = x', \\ y = y', \\ z = g^{-1}\sqrt{D(z')}\cosh gt'. \end{cases} \tag{2.12}$$

So from the inertial point of view, supported observers move on hyperbolic trajectories, with constant accelerations that depend on the observers' z'.

[12]See App. A. Throughout this chapter we use primed letters to indicate coordinates and tensors in the supported frame; also, we use D_t to denote the quantity that is simply called D in the appendix.

This is true also for supported charges, which will then radiate as discussed in Sec. 2.3.1, validating our prediction as to the result of experiment 3.

What about the converse? We obtain the trajectories of freely falling test bodies in the supported frame by means of the geodesic equation, written in supported coordinates (the required Christoffel coefficients may be found in Eq. (A.13) of App. A). Even if we can always cast the metric in the unified Rindler form (2.11) by a suitable choice of the coordinate system, we should remember that for different choices of $D(z')$ we get different shapes for the geodesics, a fact that is obviously relevant to our considerations. In particular, hyperbolic trajectories are obtained only by setting (Rohrlich, 1963)

$$D(z') = \frac{1}{\cosh^2 \sqrt{(1-gz')^2 - 1}}. \tag{2.13}$$

In other words, supported observers will not in general see a freely falling object move on a hyperbolic trajectory. As remarked by Rohrlich (1963), "this provides a simple example dispelling the often expressed belief that in general relativity acceleration is relative and therefore reciprocal in the sense that the motion of A relative to B is identical (apart from a sense of direction) with the motion of B relative to A".

2.3.3 Physics in the supported frame

Let us put our supported coordinates to a good use, by predicting the result of experiments 1 and 2. Since now the metric is not manifestly Minkowskian, Maxwell's equations have to assume their generally covariant form:

$$g'^{\alpha\beta}\nabla_\alpha\nabla_\beta A'^\mu = -4\pi j'^\mu, \tag{2.14}$$

$$F'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu \tag{2.15}$$

(as written in the Lorentz gauge $\nabla_\mu A'^\mu = 0$). To model experiment 1, we find the field of a charge at rest at $x' = y' = z' = 0$ in the supported frame. Because however we know that in the inertial frame the charge performs hyperbolic motion, we do not need to solve Eq. (2.14); instead, we can simply transform the field components (2.7) to the new coordinate system:

$$F'_{\mu\nu} = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu}F_{\alpha\beta}. \tag{2.16}$$

We then get (Rohrlich, 1963)

$$\begin{gathered} E'_{\phi'} = H'_{\rho'} = H'_{\phi'} = H'_{z'} = 0, \\ E'_{\rho'} = g(z\,E_\rho - t\,H_\phi), \qquad E'_{z'} = \frac{\mathrm{d}D(z')}{\mathrm{d}z'}\frac{E_z}{2\,g}; \end{gathered} \tag{2.17}$$

and then, using Eq. (2.7) and Eq. (2.12),

$$\begin{aligned} E'_{\rho'} &= 8\,e\,\rho' D(z')/g^3 \xi'^3, \\ E'_{z'} &= -\frac{2\,e\,(\rho'^2 + g^{-2} - g^{-2} D(z'))}{g^3 \xi'^3} \frac{\mathrm{d}D(z')}{\mathrm{d}z'}, \\ \xi' &= g^{-2}\sqrt{(1+g^2\rho'^2)^2 + 2(-1+g^2\rho'^2) D(z') + D(z')^2}. \end{aligned} \tag{2.18}$$

Because the magnetic field vanishes throughout spacetime independently of $D(z')$, it is clear by our previous discussion that the charge does not radiate in the supported frame. Thus, even if there is radiation in the inertial frame, any evidence of energy transfer is hidden to the observers who are at rest with respect to the charge. What they see instead is an electric field that can be derived from the potential

$$\phi' = eg \frac{1 + g^2\rho'^2 + D(z')}{[(1 - g^2\rho'^2 - D(z'))^2 + 4\,g^2\rho'^2]^{1/2}}. \tag{2.19}$$

The shape of the field is dependent on the choice of $D(z')$. If we write $D(z')$ up to second order as $D(z') = 1 + 2z'g + \alpha z'^2 g^2 + O(g^3)$, it follows that, up to first order in g,

$$\phi' = \frac{e}{r'} + \frac{e\,g\,z'(\rho'^2 - (\alpha - 2)z'^2)}{2\,r'^3} + O(g^2), \tag{2.20}$$

where obviously $r'^2 = \rho'^2 + z'^2$.

We can apply the same reasoning to derive the field of a freely falling charge, by transforming a pure Coulomb field to the supported frame; the transformed Coulomb field is

$$\begin{gathered} E'_{\phi'} = E'_{z'} = H'_{\phi'} = H'_{z'} = 0, \\ E'_{\rho'} = \frac{g\rho z \cdot E_r}{r}, \qquad E'_{z'} = \frac{\mathrm{d}D(z')}{\mathrm{d}z'} \frac{z \cdot E_r}{2\,g\,r}, \qquad H'_{\phi'} = -\frac{1}{D(z')} \frac{\mathrm{d}D(z')}{\mathrm{d}z'} \frac{\rho t \cdot E_r}{2\,r}; \end{gathered} \tag{2.21}$$

using $E_r = e/r^2$ and Eq. (2.12), we get

$$\begin{aligned} E'_{\rho'} &= \sqrt{D(z')} \frac{e\rho' \cosh gt'}{(\rho'^2 + g^{-2} D(z') \cosh^2 gt')^{3/2}}, \\ E'_{z'} &= \sqrt{D(z')}\, \frac{\mathrm{d}D(z')}{\mathrm{d}z'} \frac{e \cosh gt'}{2\,g^2(\rho'^2 + g^{-2} D(z')^2 \cosh^2 gt')}, \\ H'_{\phi'} &= -\frac{1}{\sqrt{D(z')}} \frac{\mathrm{d}D(z')}{\mathrm{d}z'} \frac{e\rho' \sinh gt'}{2\,g(\rho'^2 + g^{-2} D(z') \cosh^2 gt')^{3/2}}. \end{aligned} \tag{2.22}$$

In analogy to the criterion outlined in Sec. 2.3.1 for the inertial case, we can now evaluate the flux of the resulting energy–momentum tensor through the light spheres centered along the worldline of the charge (Rohrlich, 1963). The flux is not null, proving that the charge does radiate. This fact settles experiment 2 for good, and adds evidence to the claim that *the notion of radiation is not invariant with respect to transformations from inertial to accelerated frames.* Nevertheless, we still have to clarify how this noninvariance can be compatible with energy considerations.

2.3.4 The balance of energy in the four experiments

In both experiments 1 and 4 there is neither emission nor absorption of electromagnetic energy, so we are only concerned with the source of the energy transferred to the detectors in experiments 2 and 3. The discussion of Sec. 2.3.1 applies directly to experiment 3: the energy of the radiation emitted by the supported charge and detected in the inertial frame must be provided by the agency that enforces the uniformly accelerated motion of the charge. This balance is apparent in the Dirac–Lorentz equation (2.6) for all the *physical* accelerated motions, it remains true in the limit of pure hyperbolic motion.

Experiment 2 is the *direct classical analogue* of the Unruh effect: even if there is no detectable radiation in the inertial frame, the accelerated detector somehow manages to absorb energy. In the accelerated frame, this energy comes from the radiation field (2.21). Yet, from the inertial point of view, the static Coulomb field of the freely falling charge has no energy to lose. If there was no accelerating agency to support the detector, no radiation would be detected; hence, it is clear that all the absorbed energy must come from the accelerating agency. We can argue as follows: any detector of electromagnetic radiation must necessarily be charged on its own, and it must contain internal degrees of freedom. Thus the accelerating agency must supply an additional amount of work to balance the energy dissipated away by radiation reaction from the accelerating charged detector. Inertial observers will perceive this physical effect as a radiation field coming *from the detector*.

This situation parallels closely what happens in the Unruh effect, where the absorption of a Rindler particle by the accelerated detector is seen as the emission of a Minkowski particle in the inertial frame (Unruh and Wald, 1984). In the quantum case, this emission is due to the unavoidable coupling of the accelerated quantum detector to the vacuum state of the field. Surprisingly, the coupling can be shown to justify classical radiation reaction via a fluctuation–dissipation theorem (Sciama et al., 1981; Callen and Welton, 1951).

Chapter 3

Märzke–Wheeler Coordinates for Accelerated Observers in Special Relativity

3.1 Introduction

In the usual textbook special relativity, the distinction between *inertial observer* and *Lorentz coordinate frame* is blurred.[2] Because of the symmetries of Minkowski spacetime, inertial observers can label all the events of spacetime in a simple and consistent manner that is based on physical conventions and idealized procedures. [For example, inertial observers can be thought to set up Lorentz coordinate frames via a framework of ideal clocks and rigid rods that extend throughout the spacetime region of interest, outfitting it with suitable measuring devices; the clocks are synchronized with light signals; and so on. See, for instance, (Misner et al., 1973).]

For inertial observers, Lorentz coordinates are a device to extend the concept of physical reality from the observers' worldlines to the entire spacetime, building a description of the world which incorporates notions of *distance*, and *simultaneity*. Moreover, this description of physics is translated easily between inertial observers in relative motion with respect to each other, using the transformations of the Poincaré group.

It follows that in special relativity many physical notions have a joint local and global valence: they are defined with reference to the *entire* Minkowski spacetime, but they also carry a well defined meaning for *lo-*

[1]Originally published as M. Pauri and M. Vallisneri, *Found. Phys. Lett.*, in press (October 2000). gr-qc/0006095.

[2]We will have more to say about this in Sec. 4.3.1.

cal inertial observers. An instance is the notion of *particle* in quantum field theory, which corresponds to a *global*, quantized classical mode of the field extending across Minkowski spacetime, but also to the outcome of *local* detections along an observer's trajectory (we discussed this at length in Ch. 2).

Now, suppose we are interested in the observations made by *noninertial* observers: of course we could study their physics in some given *laboratory* inertial frame of reference. Yet if we could rewrite all equations in a set of coordinates that is somehow *adapted and natural* to the observers' accelerated motion, we would obtain an interesting representation of the *intrinsic* physics that the accelerated observers experience. A well known example is the Unruh effect (Unruh, 1976), where laboratory physics predicts that a uniformly accelerated observer moving through the Minkowski quantum vacuum will behave as if in contact with a thermal bath, whereas intrinsic physics describes the Minkowski vacuum as consisting of a thermal distribution of quantum particles (under a notion of particle appropriate to the the accelerated observer).

3.2 Definition of coordinates for accelerated observers

We set out to define an adapted coordinate system for an accelerated observer (we shall call him *Axel*) who is moving through Minkowski spacetime. Since the accelerated system should describe Axel's intrinsic physics, its time coordinate should coincide with Axel's proper time. Moreover, around any event of Axel's worldline, there is a small neighborhood where the accelerated coordinates should approximate Axel's instantaneous Lorentz rest frame. To satisfy these requirements, we can propagate a *Fermi–Walker transported tetrad*[3] along the worldline, and use the tetrad vectors (one of which will point along Axel's four-velocity) to reach out to Axel's surroundings.

How do we extend these prescriptions to cover the entire Minkowski spacetime? We can define *extended-tetrad coordinates* by stretching out rigidly the Fermi–Walker transported axes beyond Axel's immediate vicinity, but we run into trouble soon: for instance, if Axel (a) starts at rest, (b)

[3]Fermi–Walker transported vectors "change from instant to instant by precisely that amount implied by the change of the four-velocity" (Misner et al., 1973, p. 170); the transported four-vectors for Axel then obey

$$\frac{dv^\mu}{d\tau} = (u^\mu a^\nu - u^\nu a^\mu)v_\nu, \tag{3.1}$$

where τ is Axel's proper time, $u^\mu = dx^\mu/d\tau$ his four-velocity, and $a^\mu = du^\mu/d\tau$ his acceleration.

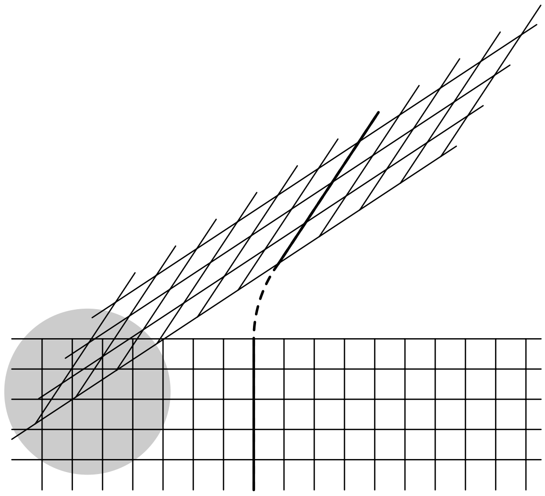

Figure 3.1: Worldline of an observer who undergoes a brief period of acceleration (shown dashed). The extension of the *Fermi–Walker transported* coordinate system runs into trouble when different constant-time surfaces overlap on the left. [Adapted from (Misner et al., 1973).]

moves for a while with constant acceleration $|a| = g$, then (c) continues with constant velocity (see Fig. 3.1), we find that the constant-time surfaces of phase a overlap with those of phase c, at a distance of order g^{-1} from Axel's worldline. The constant-time planes intersect because they are orthogonal to Axel's four-velocity u^μ, which tilts during accelerated motion.

3.3 The Märzke–Wheeler procedure

We need a way to *foliate* Minkowski spacetime into *nonoverlapping surfaces of simultaneity* that are adapted to Axel's motion and that reduce to local Lorentz frames around his worldline. Märzke and Wheeler (1964) discussed an extension of *Einstein's synchronization convention*[4] to synchronize observers in curved spacetime. The notion of Märzke–Wheeler simultaneity, *restricted to accelerated observers in flat spacetime*, has just the properties we need.[5] We use it to build *Märzke–Wheeler coordinates*, specified as fol-

[4]By Einstein's *convention*, two *inertial* observers get synchronized by exchanging light signals, while assuming that the one-way speed of light between the inertial worldlines is equal to the average round-trip speed. The resulting notion of simultaneity yields the standard slicing of Minkowski spacetime into hyperplanes of constant Lorentz coordinate time. In Ch. 4 we will have much to say about the *conventionality* of Einstein and Märzke–Wheeler simultaneity.

[5]Our construction bears resemblance to some applications of Milne's *k-calculus* (Page, 1936) and to other arguments in the literature (Ives, 1950; Whitrow, 1961;

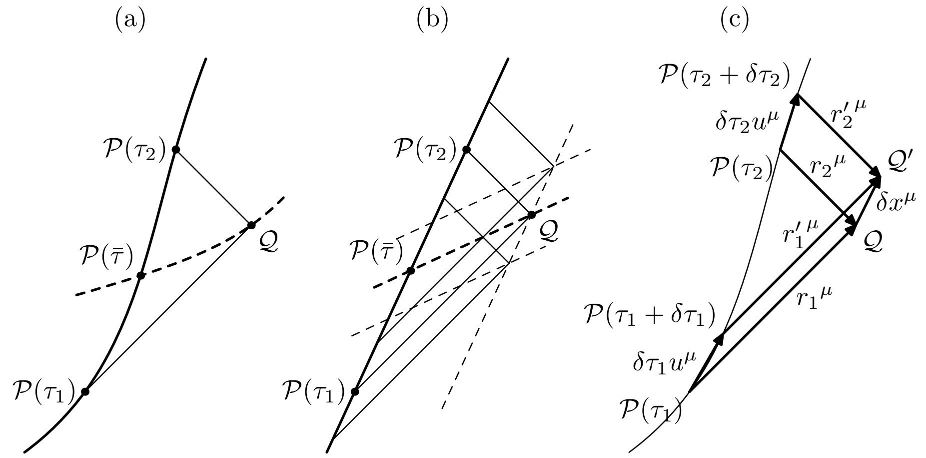

Figure 3.2: Definition of Märzke–Wheeler coordinates. (a): General case. (b): Inertial case. Märzke–Wheeler coordinates reproduce a Lorentz frame. (c): Proof that the constant-$\bar{\tau}$ surfaces are spacelike (see p. 27).

lows. Imagine that: (a) at each event along his worldline $\mathcal{P}(\tau)$, accelerated observer Axel emits a flash of light imprinted with his proper time; (b) in the spatial region that Axel wants to monitor, there are labeling devices capable of receiving Axel's flashes and of sending them back with their signature; (c) Axel is always on the lookout for returning signals (see Fig. 3.2a). Now, suppose that, at the event $\mathcal{Q}$, a labeling device receives and rebroadcasts a light flash originally emitted by Axel at proper time τ_1, and that Axel receives the returning signal at proper time τ_2. Then Axel will *conventionally* label $\mathcal{Q}$ with a time coordinate $\bar{\tau} = (\tau_1 + \tau_2)/2$ and with a radial coordinate $\sigma = (\tau_2 - \tau_1)/2$. These two coordinates can then be completed by two angular coordinates which specify the direction of $\mathcal{Q}$ with respect to $\mathcal{P}(\bar{\tau})$.

If $\mathcal{P}(\tau)$ is an inertial worldline, the constant-$\bar{\tau}$ surfaces are just constant–Lorentz-time surfaces, and σ is simply the radius of $\mathcal{Q}$ in spherical Lorentz coordinates (see Fig. 3.2b): for inertial observers, Märzke–Wheeler coordinates reduce to Lorentz coordinates (see App. D for a proof in a special case). Even better, this procedure yields well defined coordinates $\bar{\tau}$ and σ *for any event* $\mathcal{Q}$ *that lies in the intersection of the causal past and causal future*[6] *of the worldline* $\mathcal{P}(\tau)$ (we shall refer to this set as the *causal envelope* of $\mathcal{P}(\tau)$; it contains all the events from which bidirectional communication with Axel is possible). Proof: (a) the past and future lightcones of $\mathcal{Q}$ necessarily intersect with $\mathcal{P}(\tau)$ somewhere, by definition of causal future and

Kilmister and Tonkinson, 1993).

[6]See (Wald, 1984, Ch. 8) for these and other definitions concerning the causal structure of spacetime.

past; (b) the intersection of a null surface with a timelike curve is unique, so once $\mathcal{Q}$ is given, τ_1 and τ_2 are well defined. It follows also that constant-$\bar{\tau}$ surfaces cannot intersect.

We shall use the notation $\Sigma_{\bar{\tau}}$ to refer to the surface of simultaneity labeled by the Märzke–Wheeler time $\bar{\tau}$. To prove that each $\Sigma_{\bar{\tau}}$ is spacelike, refer to Fig. 3.2c, and consider a point $\mathcal{Q}'$ that is displaced infinitesimally from $\mathcal{Q}$; the future lightcone with origin in $\mathcal{Q}'$ intersects Axel's worldline at the event $\mathcal{P}(\tau_2 + \delta\tau_2)$. Define

$$\begin{aligned} {r_2}^\mu &\equiv \big(\mathcal{Q} - \mathcal{P}(\tau_2)\big)^\mu, \\ {r'_2}^\mu &\equiv \big(\mathcal{Q}' - \mathcal{P}(\tau_2 + \delta\tau_2)\big)^\mu, \\ \delta x^\mu &\equiv \big(\mathcal{Q}' - \mathcal{Q}\big)^\mu; \end{aligned} \tag{3.2}$$

both ${r_2}^\mu$ and ${r'_2}^\mu$ are null vectors. Since the displacements are infinitesimal, we can write

$$\big(\mathcal{P}(\tau_2 + \delta\tau_2) - \mathcal{P}(\tau_2)\big)^\mu = \delta\tau_2\, u^\mu(\tau_2) \tag{3.3}$$

(u^μ is Axel's four-velocity). Then we have

$$\begin{aligned} 0 = |{r'_2}^\mu|^2 &= |{r_2}^\mu + \delta x^\mu - \delta\tau_2\, u^\mu|^2 = \\ &= |{r_2}^\mu|^2 + 2\, {r_2}^\mu(\delta x_\mu - \delta\tau_2\, u_\mu) + O(\delta\tau^2) = \\ &= 2{r_2}^\mu(\delta x_\mu - \delta\tau_2\, u_\mu) + O(\delta\tau^2), \end{aligned} \tag{3.4}$$

and

$$\frac{\partial\tau_2}{\partial x^\mu} = \frac{r_{2\mu}}{{r_2}^\nu u_\nu(\tau_2)}. \tag{3.5}$$

The same relation holds for $\partial\tau_1/\partial x^\mu$:

$$\frac{\partial\tau_1}{\partial x^\mu} = \frac{r_{1\mu}}{{r_1}^\nu u_\nu(\tau_1)}, \tag{3.6}$$

where ${r_1}^\mu \equiv (\mathcal{Q} - \mathcal{P}(\tau_1))^\mu$. So we can write the normal vector to the constant-$\bar{\tau}$ surface as

$$\frac{\partial\bar{\tau}}{\partial x^\mu} = \frac{1}{2}\left(\frac{\partial\tau_1}{\partial x^\mu} + \frac{\partial\tau_2}{\partial x^\mu}\right) = \frac{1}{2}\left(\frac{r_{1\mu}}{{r_1}^\nu u_\nu(\tau_1)} + \frac{r_{2\mu}}{{r_2}^\nu u_\nu(\tau_2)}\right). \tag{3.7}$$

Furthermore,

$$\left|\frac{\partial\bar{\tau}}{\partial x^\mu}\right|^2 = \frac{{r_1}^\mu r_{2\mu}}{\big({r_1}^\nu u_\nu(\tau_1)\big)\,\big({r_2}^\nu u_\nu(\tau_2)\big)}. \tag{3.8}$$

Looking at Fig. 3.2c, you can convince yourself that ${r_1}^\mu r_{2\mu} > 0$, ${r_1}^\nu u_\nu(\tau_1) > 0$, and ${r_2}^\nu u_\nu(\tau_2) < 0$ (throughout this chapter we set $c = 1$ and take a timelike metric). Consequently, the surfaces of constant-$\bar{\tau}$ have normal vectors that are timelike everywhere. Under appropriate hypotheses of smoothness for the worldline $\mathcal{P}(\tau)$, the constant-$\bar{\tau}$ surfaces will also be differentiable; altogether, they qualify as spacelike.

Whereas the constant-time surfaces obtained by the extended-tetrad procedure (described in Sec. 3.2) are always three-dimensional planes, *the global shape of the Märzke–Wheeler constant-$\bar{\tau}$ surfaces depends on the entire history of the observer, both past and future.*[7] Accordingly, the three-dimensional metric ${}^3g_{ij}$ induced by the Minkowski metric on the surfaces will depend on $\bar{\tau}$. This is true in general, but not for *stationary worldlines*, defined by

$$\forall\tau,\ |\mathcal{P}(\tau+\Delta\tau)-\mathcal{P}(\tau)| = |\mathcal{P}(\tau)-\mathcal{P}(0)|. \tag{3.9}$$

Stationary worldlines represent motions that show the same behavior at all proper times. Synge (1967) and Letaw (1981) obtained stationary worldlines by the alternative definition of relativistic trajectories with constant acceleration and curvatures. In App. B, we briefly review their classification, as given by Synge (1967). For stationary worldlines, the surfaces $\Sigma_{\bar{\tau}}$ maintain always the same shape and metric.

You can easily build a stationary trajectory by taking any timelike integral curve of the isometries of Minkowski spacetime, and rescaling its parametrization to obtain a worldline that satisfies $u^\mu u_\mu = -1$. Indeed, in this way we can obtain *any* stationary trajectory, because we can always write its four-velocity as a linear combination U^μ of the ten Minkowski Killing fields[8] (i. e., the infinitesimal generators of isometries). The simplest case of stationary trajectories are inertial worldlines, obtained by combining the Killing fields of a time translation and a space translation; further examples are linear uniform acceleration and uniform rotation, obtained as the integral curves of, respectively, a Lorentz boost and a rotation plus a time translation.

No matter how we choose to define the constant-time surfaces of a stationary observer (call her *Stacy*), the Killing field U^μ (which coincides with u^μ on Stacy's worldline, but is defined all over Minkowski spacetime) generates infinitesimal translations in time that carry each constant-time surface into the next one, while conserving its three-metric. Once Stacy has chosen a *single* constant-time surface and a set of spatial coordinates to describe it, she can use U^μ to propagate the surface and its coordinates forward and backward in time, defining coordinates for the entire Minkowski spacetime.

[7] Yet this global dependence is hierarchical. Take for instance the constant-time surface $\bar{\tau} = \tau_0$, with origin in $\mathcal{P}(\tau_0)$: the behavior of the worldline at proper times that lie to the future of $\tau_0 + \Delta\tau$, or to the past of $\tau_0 - \Delta\tau$, can only influence the structure of the constant-time surface for $\sigma > \Delta\tau$.

[8] They are the four translations ∂_t, ∂_x, ∂_y, ∂_z, the three boosts $x\,\partial_t + t\,\partial_x$, $y\,\partial_t + t\,\partial_y$, $z\,\partial_t + t\,\partial_z$, and the three rotations, $y\,\partial_z - z\,\partial_y$, $z\,\partial_x - x\,\partial_z$ $x\,\partial_y - y\,\partial_x$.

3.4 Märzke–Wheeler coordinates for stationary observers

Stationary curves are a very useful arena to compare Märzke–Wheeler coordinates with other accelerated systems, such as the stationary coordinates derived by Letaw and Pfautsch (1982). As a first example, suppose Stacy moves with linear, uniform acceleration in (1+1)-dimensional Minkowski spacetime[9] (so she will be *Hyper-Stacy*). We can write her trajectory as

$$\begin{cases} t = g^{-1} \sinh g\tau, \\ x = g^{-1} \cosh g\tau, \end{cases} \quad (\textit{Hyper-Stacy}\text{: worldline}) \tag{3.10}$$

which is an integral curve of the infinitesimal Lorentz boost $U^\mu = g(x\,\partial_t + t\,\partial_x)$, where g is the magnitude of the acceleration. In this case, the extended-tetrad procedure gives the traditional Rindler coordinates (Rindler, 1975):

$$\begin{cases} t &= g^{-1}(1+\xi) \sinh g\tau, \\ x &= g^{-1}(1+\xi) \cosh g\tau. \end{cases} \quad (\textit{Hyper-Stacy}\text{: Rindler coordinates}) \tag{3.11}$$

You can check easily that the flow of U^μ carries the constant-τ surfaces backward and forward in τ, and that the *Rindler metric* $ds^2 = -(1+g\xi)^2 d\tau^2 + d\xi^2$ is always conserved. Let us now derive Märzke–Wheeler coordinates for Hyper-Stacy's motion. According to our prescriptions, the surface $\Sigma_{\bar\tau=0}$ [the set of the events that are simultaneous to $\mathcal{P}(0)$] includes all the events that, for some σ, receive light signals from $\mathcal{P}(-\sigma)$ and send them back to $\mathcal{P}(\sigma)$. By symmetry, $\Sigma_{\bar\tau=0}$ must coincide with the positive-x semiaxis; we then find that the Märzke–Wheeler radial coordinate is $\sigma = g^{-1} \log gx$. Using the finite isometry generated by U^μ with parameter $\bar\tau'$, we can now turn $\Sigma_{\bar\tau=0}$ into any other $\Sigma_{\bar\tau'}$. Altogether, the coordinate transformation between Minkowski and Märzke–Wheeler coordinates is

$$\begin{cases} t &= g^{-1} e^{g\sigma} \sinh g\bar\tau, \\ x &= g^{-1} e^{g\sigma} \cosh g\bar\tau. \end{cases} \quad (\textit{Hyper-Stacy}\text{: M.–W. coordinates}) \tag{3.12}$$

The Rindler and Märzke–Wheeler constant-time surfaces coincide, and indeed the two coordinate sets are very similar. (If we identify ξ with σ and τ with $\bar\tau$, they coincide up to linear order, because both systems must coincide with local Lorentz frames in the vicinity of the worldline.)

We turn now to a more interesting example, where Märzke–Wheeler coordinates show a much richer structure than expected by conventional wisdom: uniform relativistic rotation.[10] A typical trajectory in 2+1 dimensions

[9] This is the *hyperbolic motion* that we first encountered in Sec. 2.3.1. In Synge's classification (1967), it is a *type-IIa helix*.

[10] In Synge's classification (1967), a *type-IIc helix*.

for *Roto-Stacy* (who else?) is

$$\begin{cases} t &= \sqrt{1+R^2\Omega^2}\,\tau, \\ r &= R, \\ \phi &= \Omega\,\tau, \end{cases} \qquad (\textit{Roto-Stacy}\text{: worldline}) \tag{3.13}$$

where the constant R is the geometric radius of the trajectory, and Ω is the proper angular velocity; the coordinate angular velocity is $d\phi/dt = \Omega/\sqrt{1+\Omega^2 R^2}$. Finally, Roto-Stacy's generating Killing vector field is $U^\mu = \sqrt{1+R^2\Omega^2}\,\partial_t + \Omega\,\partial_\phi$. The traditional coordinate system for Roto-Stacy are rigidly rotating coordinates:

$$\begin{cases} t &= \sqrt{1+R^2\Omega^2}\,\tau, \\ r &= r', \\ \phi &= \phi' + \Omega\,\tau \end{cases} \qquad (\textit{Roto-Stacy}\text{: rigidly rotating coordinates}) \tag{3.14}$$

(some authors even define $t = \tau$, violating the first requirement we set in Sec. 3.2). In these coordinates, Roto-Stacy stands fixed in space at $r' = R$, $\phi' = 0$; the constant-τ surfaces coincide with constant-t planes; and the points with fixed r' and ϕ' rotate in the inertial frame with angular velocity $d\phi/dt = \Omega/\sqrt{1+\Omega^2 R^2}$, which is faster than light for $r' > r'_{\text{lim}} = \sqrt{1+\Omega^2 R^2}/\Omega^2$. The metric is

$$\begin{aligned} ds^2 &= -\,(1+\Omega^2 R^2)\,d\tau^2 + r'^2(d\phi' + \Omega\,d\tau)^2 + dr'^2 = \\ &= -\,[1 + (R^2 - r'^2)\,\Omega^2]\,d\tau^2 + 2\,\Omega\,r'^2\,d\tau\,d\phi' + r'^2 d\phi'^2 + dr'^2. \end{aligned} \tag{3.15}$$

(*Roto-Stacy*: rigidly rotating metric)

Now move on to Märzke–Wheeler coordinates, and consider at first the surface $\Sigma_{\bar\tau=0}$. Märzke–Wheeler coordinates have their origin at Roto-Stacy's position, $\mathcal{P}(0)$: $(x = R,\, y = 0)$. We find the curves of constant σ as the intersection (an ellipse) of the future lightcone of $\mathcal{P}(-\sigma)$ with the past lightcone of $\mathcal{P}(\sigma)$. As σ increases, the ellipses move outward, weaving the surface $\Sigma_{\bar\tau=0}$, which turns out to be defined by

$$\begin{cases} t &= c(\sigma)\sin\theta, \\ x &= b(\sigma)\cos\theta + R\cos\Omega\sigma, \\ y &= a(\sigma)\sin\theta, \end{cases} \quad (\textit{Roto-Stacy}\text{: M.–W. coord., } \bar\tau = 0) \tag{3.16}$$

where $a(\sigma) = \sqrt{1+R^2\Omega^2}\,\sigma$, $c(\sigma) = R\sin\Omega\sigma$, and $b(\sigma) = \sqrt{a^2(\sigma) - c^2(\sigma)}$ (see App. C; our choice of the angular coordinate is conventional, but convenient). As σ increases, the centers of the ellipses oscillate on the x axis between R and $-R$; the semiaxes $a(\sigma)$ and $b(\sigma)$ grow in such a way that no two ellipses ever intersect; and the ellipses themselves pitch up and down in

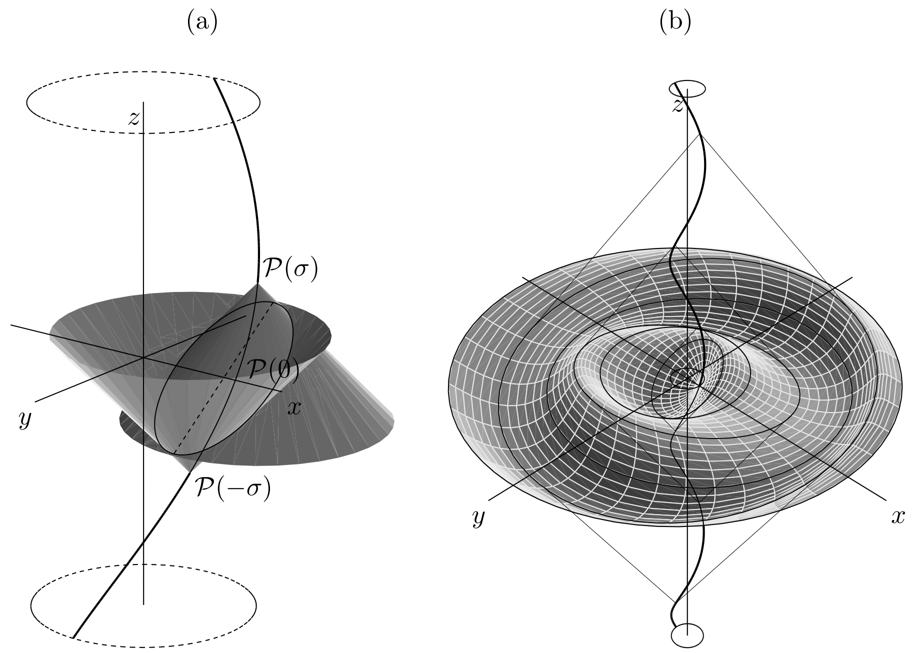

Figure 3.3: Geometry of constant-$\bar{\tau}$ surfaces for uniformly rotating observers. (a): The intersection of the lightcones with origin in $\mathcal{P}(-\sigma)$ and $\mathcal{P}(\sigma)$ defines an ellipse. (b): The union of all constant-σ ellipses weaves the constant-$\bar{\tau}$ surface. Notice the oscillating pitch of the ellipses.

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the time direction, as if they were hinging on the y axis (see Fig. 3.3), so the Märzke–Wheeler constant-$\bar{\tau}$ surface $\Sigma_{\bar{\tau}=0}$ deviates in undulatory fashion with respect to the Minkowski constant-time surface $t = 0$ [because any event $\mathcal{Q}$ looks closer when the emission event $\mathcal{P}(-\sigma)$ and the reception event $\mathcal{P}(\sigma)$ are on the near side of the origin; it looks farther if they are on the other side]. In the limit $\sigma \to \infty$, the constant-σ ellipses turn into circles; but the undulation in the t direction maintains the finite amplitude R.

We use the isometry generated by U^μ to propagate these coordinates from $\Sigma_{\bar{\tau}=0}$ throughout Minkowski spacetime. The complete transformation between Minkowski and Märzke–Wheeler coordinates is then

$$\begin{cases} t = c(\sigma)\sin\theta + \sqrt{1+R^2\Omega^2}\,\tau, \\ \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\Omega\tau & -\sin\Omega\tau \\ \sin\Omega\tau & \cos\Omega\tau \end{pmatrix} \cdot \begin{pmatrix} b(\sigma)\cos\theta + R\cos\Omega\sigma \\ a(\sigma)\sin\theta \end{pmatrix}. \end{cases} \tag{3.17}$$

3.5 Märzke–Wheeler coordinates and the paradox of the twins

Märzke–Wheeler coordinates cast a new light on the relativistic *paradox of the twins*.[11] This *gedankenexperiment* earns the designation of *paradox* because, at first sight, the motion of the twins is reciprocal, whereas the physical effects of relativistic time dilation are not. In the usual arrangement, shown in Fig. 3.4a, the journeying twin (*Ulysses*) moves with constant speed v, first away from and then toward the waiting, inertial (and nonidentical!) twin *Penelope*. According to the Lorentz transformation between the inertial frames associated with the twins, Penelope sees Ulysses' proper time as dilated by the relativistic factor $\gamma = (1 - v^2)^{-1/2} > 1$, so when the twins are rejoined, Penelope has aged γ times more than Ulysses. Yet, it is also true that Penelope always moves with a speed v relatively to Ulysses, so *he* should see *her* proper time as dilated, and *he* should be older in the end!

The problem is that the notion of time dilation, as it is usually discussed, amounts to little more than a statement on how to relate the coordinate times of different Lorentz frames; it also concerns the observations of different inertial observers, whose proper times coincide with the coordinate times of their Lorentz rest frames. Now, Ulysses is *not* an inertial observer *throughout* his motion, because at event $\mathcal{C}_U$ he turns around and begins his return trip toward Penelope. Along the worldline segments $\mathcal{A}\mathcal{C}_U$ and $\mathcal{C}\mathcal{B}_U$, it is correct to say that Ulysses sees Penelope's proper time as dilated, in the following sense: if Ulysses compares his proper time with Penelope's at events which are simultaneous in his Lorentz rest frame, then Penelope appears to be aging at a slower pace. However, when Ulysses inverts his velocity at $\mathcal{C}_U$ (see Fig. 3.4b), he switches to a new Lorentz frame, and his constant-time surfaces change their spacetime orientation abruptly. Just before arriving in $\mathcal{C}_U$, Ulysses considers himself simultaneous to the event $\mathcal{C}'_P$ along Penelope's worldline; just after leaving $\mathcal{C}_U$, according to his new Lorentz frame, Ulysses considers himself simultaneous to $\mathcal{C}''_P$. However, $\mathcal{C}'_P$ and $\mathcal{C}''_P$ are distinct events, separated by a finite lapse of time! *There is a finite section of Penelope's worldline which Ulysses effectively skips and to which he is never simultaneous.* Because of this missing finite lapse of Penelope's proper time, Ulysses is younger at his final reunion with Penelope, even if throughout the trip he reckoned that Penelope was aging at a slower pace than him![12]

[11] The literature on the subject is immense and often redundant. Even if the paradox was already present in Einstein's 1905 seminal paper, it was P. Langevin who first presented it in its modern form. Arzeliès (1966) and Marder Marder (1971) give excellent annotated bibliographies for contributions up to, respectively, 1966 and 1971.

[12] Ulysses' switch of Lorentz frames in $\mathcal{C}_U$ has generated some controversy, centered on the physical effects of Ulysses' acceleration around $\mathcal{C}_U$. These effects are irrelevant, as can be seen by the *third twin argument* introduced by Lord Halsbury (Salmon, 1975): in brief,

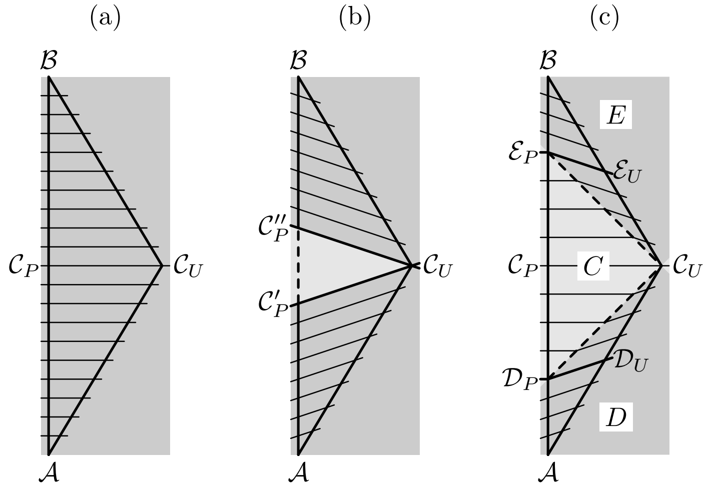

Figure 3.4: The relativistic *paradox of the twins*.

The worldlines of the twins are drawn in the Lorentz rest frame of the inertial twin, Penelope, who moves in spacetime from $\mathcal{A}$ to $\mathcal{B}$ through $\mathcal{C}_P$. The journeying twin, Ulysses, travels first from $\mathcal{A}$ to $\mathcal{C}_U$ with velocity v, then inverts his motion to rejoin Penelope in $\mathcal{B}$.

(a) Lorentz slicing of spacetime according to Penelope.

(b) Lorentz slicing of spacetime according to Ulysses, on his separate stretches of inertial motion. Ulysses skips a finite lapse of Penelope's worldline (shown dashed).

(c) Märzke–Wheeler slicing of spacetime, according to Ulysses. This slicing coincides with the Lorentz slicing in b for events in the regions D and E (these events belong to the causal envelopes of the worldline segments $\mathcal{A}\mathcal{C}_U$ and $\mathcal{C}\mathcal{B}_U$), but it shows a peculiar structure in region C.

From a general-relativistic perspective, there is no paradox from the beginning: Ulysses and Penelope move on different spacetime paths between the same two events. The lapse of proper time is a particular functional of the path followed: no wonder that it is different for the two twins! The surprise of nonreciprocal time dilation arises because (a) Ulysses needs to compare *simultaneous* events on his and on Penelope's worldlines to know who is aging faster, so he needs a global notion of simultaneity or, equivalently, a slicing of spacetime into spacelike, constant-time surfaces; (b) because our Ulysses has a special-relativistic background, he naturally em-

at $\mathcal{C}_U$ Ulysses communicates the reading of his clock to a third twin who was already traveling toward Penelope with velocity v; thus, the proper time elapsed on the paths $\mathcal{A}\mathcal{C}_U\mathcal{B}$ and $\mathcal{A}\mathcal{C}_P\mathcal{B}$ can be compared without any twin ever experiencing acceleration.

ploys the slicing implicit in his two distinct Lorentz rest frames; (c) but that slicing fails to cover a finite region of spacetime, where nevertheless Penelope spends part of her time!

Märzke–Wheeler coordinates avoid this problem, because by definition they provide a consistent time slicing of the causal envelope of any observer's worldline: Ulysses and Penelope stay well inside each other's causal envelope, simply because they start together and cannot travel faster than light. For inertial Penelope, Märzke–Wheeler coordinates reproduce a Lorentz rest frame (Fig. 3.4a). So nothing changes in her account of Ulysses' aging: her proper time lapse Δt_P is γ times Ulysses' proper time lapse Δt_U.

Likewise, Märzke–Wheeler coordinates for Ulysses do reproduce a Lorentz frame, but only and separately for the events in the causal envelopes (D and E) of the segments of Ulysses' uninterrupted inertial motion ($\mathcal{AC}$ and $\mathcal{CB}$; see Fig. 3.4c). In the process of Märzke–Wheeler synchronization, the events in D and E communicate with events along the *same* segment. On the contrary, region C contains events that are spacelike related to $\mathcal{C}$, and that receive light signals from $\mathcal{AC}$ and reflect them back to $\mathcal{CB}$. It is in this region that the noninertial character of Ulysses' motion becomes manifest. A simple calculation (App. D) yields the slicing structure shown in Fig. 3.4c: in D and E the slices assume the typical inclination of Lorentz constant-time surfaces, but in C the slices become perpendicular to $\mathcal{AB}$ (Penelope's worldline), because they split the difference between the two opposing inertial motions $\mathcal{AC}$ and $\mathcal{CB}$.

If Ulysses employs the Märzke–Wheeler notion of simultaneity to compare his age with Penelope's at simultaneous times, he accounts for the final aging difference as follows. As long as Penelope's trajectory remains within the regions D and E where the Märzke–Wheeler and Lorentz notions of simultaneity coincide, Ulysses ages γ times faster than Penelope, just as a naïve use of relativistic time dilation would imply. However, when Penelope moves through region C (from $\mathcal{D}_P$ to $\mathcal{E}_P$), she ages $\gamma(1 + v) > 1$ times faster than Ulysses (who moves from $\mathcal{D}_U$ to $\mathcal{E}_U$). Altogether, when the twins are rejoined in $\mathcal{B}$, Ulysses is younger by an overall factor of γ. See Table 22 and Fig. 3.5 for a precise tally of proper times. In App. E we study Ulysses' Märzke–Wheeler interpretation of Penelope's aging in a modified construction where Ulysses moves with constant speed and acceleration on *Roto-Stacy*'s circular trajectory. The resulting $t_P[t_U]$ (Fig. E.1) is smooth and resembles qualitatively the function shown in Fig. 3.5.

Keep in mind that the comparison of local relative aging is dependent on how we slice spacetime into constant-time surfaces. Alternative slicings will lead Ulysses to *different distributions* of Penelope's total proper time along his worldline. Stautberg Greenwood (1976) defines simultaneity by integrating the Doppler-shifted frequency of monocromatic signals exchanged by the twins. Unruh (1981) employs the notion of *parallax distance* to extend

	Ulysses' Δt_U in segment	Ulysses' total t_U	Penelope's Δt_P in segment	Penelope's total t_P	$\frac{dt_P}{dt_U}$ in segment
$\mathcal{AD}$	$\frac{1}{2(1+v)}$	$\frac{1}{2(1+v)}$	$\frac{1}{2}\frac{1-v}{\sqrt{1-v^2}}$	$\frac{1}{2}\frac{1-v}{\sqrt{1-v^2}}$	$\sqrt{1-v^2}$
$\mathcal{DC}$	$\frac{v}{2(1+v)}$	$\frac{1}{2}$	$\frac{1}{2}\frac{v}{\sqrt{1-v^2}}$	$\frac{1}{2}\frac{1}{\sqrt{1-v^2}}$	$\frac{1+v}{\sqrt{1-v^2}}$
$\mathcal{CE}$	$\frac{v}{2(1+v)}$	$\frac{1+2v}{2(1+v)}$	$\frac{1}{2}\frac{v}{\sqrt{1-v^2}}$	$\frac{1}{2}\frac{1+v^2}{\sqrt{1-v^2}}$	$\frac{1+v}{\sqrt{1-v^2}}$
$\mathcal{EB}$	$\frac{1}{2(1+v)}$	1	$\frac{1}{2}\frac{1-v}{\sqrt{1-v^2}}$	$\frac{1}{\sqrt{1-v^2}}$	$\sqrt{1-v^2}$

Table 3.1: Evolution of Ulysses' and Penelope's proper times along the segments shown in Fig. 3.4c.

All comparisons are made at events that are simultaneous according to Ulysses' Märzke–Wheeler slicing. The last column shows that Ulysses' ages faster than Penelope's along segments $\mathcal{AD}$ and $\mathcal{EB}$, but not along $\mathcal{DC}$ and $\mathcal{CE}$. Units are normalized so that Ulysses' total proper time lapse is 1.

Ulysses' local definitions of space and time, to the effect that at times he sees Penelope recede in time. Debs and Redhead (1996) analyze the class of slicings induced by Reichenbach's *nonstandard synchronies* (Redhead, 1993), which generalize the Einstein convention by positing different speeds for the light signals in the two directions.[13] However, we believe that Märzke–Wheeler slicing has a simple physical rationale and that it does a good job of locating the nonreciprocal, differential aging in the region of spacetime where the nonlocal effects of Ulysses' turnaround in $\mathcal{C}$ are felt.

3.6 Märzke–Wheeler coordinates and the Unruh effect

We wish to suggest yet another application for Märzke–Wheeler coordinates. As we underlined in Sec. 2.2, the well-known Unruh effect (Unruh, 1976) has two mutually supporting but fundamentally distinct derivations. In its *laboratory* version (in the sense introduced in Sec. 3.1), a quantum monopole detector (DeWitt, 1979), moving on a uniformly accelerated, classical worldline through the Minkowski quantum vacuum, is excited with the same probability of an *inertial* detector in contact with a thermal bath of particles. In the *intrinsic* version, free-field quantization is carried out in Rindler coordinates, which are regarded as adapted to uniformly accelerated observers: with some caution (see Note 4 on p. 7), the vacuum state of the standard Minkowski quantum field theory can be translated into a state of the accelerated theory; as it turns out, the translated state describes a thermal distribution of Rindler particles. In both the laboratory and the in-

[13]We shall examine nonstandard synchrony closely in Ch. 4.

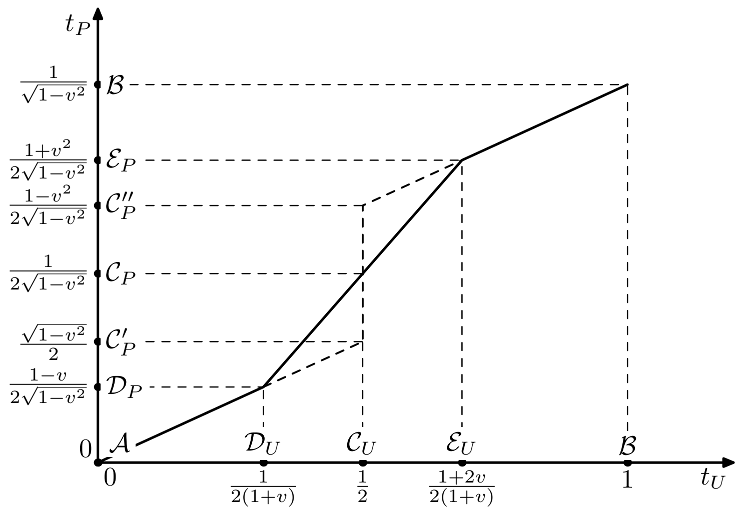

Figure 3.5: Penelope's proper time, in units of Ulysses' total proper-time lapse, as determined by Ulysses with Lorentz slicing (dashed line, see Fig. 3.4b) or Märzke–Wheeler slicing (continuous line, see Fig. 3.4c).

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trinsic versions, the temperature of the thermal bath is directly proportional to the acceleration of the detector (or of the observer).

If we try to generalize the Unruh effect from uniformly accelerated to other stationary observers, we soon run into an inconsistency between laboratory and intrinsic physics (Letaw, 1980, 1981; Letaw and Pfautsch, 1981; Davies et al., 1996; Vallisneri, 1997). For instance, laboratory physics predicts that a quantum monopole detector in uniform circular motion through the Minkowski quantum vacuum will be excited (its excitation probability will be consistent with a non–Planckian particle distribution, and it will depend on the acceleration but also on the curvature of the worldline). However, if we use the rigidly rotating coordinates of Eq. (3.14) to obtain an intrinsic quantization of the field, we merely translate the Minkowski vacuum state into another vacuum state. In short, for observers and detectors on some types of stationary worldlines, the intrinsic version of the Unruh effect is blind to the particles that are seen in the laboratory version.

We note here that the extension of the Unruh effect to worldlines even more general than stationary worldlines is not usually discussed in the literature, for two related reasons. From the viewpoint of laboratory physics, it is only for stationary worldlines that proper time factors out of the detector excitation equations, so we can turn the time-integrated excitation probability into a constant probability *rate* along the worldline. From the intrinsic viewpoint, it is only for stationary worldlines that spacetime can be easily foliated into a succession of identical spatial surfaces, mapped into each other by a Killing vector field. This property is desirable to extract the time dependence of the classical wave equation, and to define quantum particles from a set of classical wave modes that maintain the same structure on successive spacelike slices.

Let us go back to the stationary trajectories for which field quantization in adapted coordinates fails to reproduce the measurements of accelerated detectors. Some authors (Letaw and Pfautsch, 1981; Davies et al., 1996) have blamed the failure on the fact that, under the definition of time induced by the stationary Killing field, the set of *positive frequency* classical modes (which are picked up by the accelerated detector) is not the same as the set of *positive norm* modes (which determine the particle content of the vacuum fluctuations, as seen in the accelerated frame). This discrepancy seems to occur when the Killing field is not timelike everywhere: for rotating observers and quantization in rigidly rotating coordinates, it happens at radii $R > \Omega^{-1}$, where the tangential velocity of the rigid system exceeds the speed of light.

We have found an argument that suggests that field quantization in the Märkze–Wheeler coordinates of stationary observers might reproduce the observations of accelerated detectors, solving the contradiction between the laboratory and intrinsic versions of the Unruh effect. Let us see how. In the simple case of a massless Klein–Gordon quantum field, the excitation probability of the detectors can be written as (see, for instance, Vallisneri, 1997)

$$\begin{aligned} R(\omega) &= \int_{-\infty}^{+\infty} \mathrm{d}\tau\, e^{-i\omega\tau} \langle 0 | \hat{\phi}\big(\mathcal{P}(0)\big) \hat{\phi}\big(\mathcal{P}(\tau)\big) | 0 \rangle = \\ &= \int_{-\infty}^{+\infty} \mathrm{d}\tau\, e^{-i\omega\tau} \langle 0 | \hat{\phi}\big(\mathcal{P}(-\tau/2)\big) \hat{\phi}\big(\mathcal{P}(\tau/2)\big) | 0 \rangle = \\ &= 2 \int_{-\infty}^{+\infty} \mathrm{d}\sigma\, e^{-2i\omega\sigma} \langle 0 | \hat{\phi}\big(\mathcal{P}(-\sigma)\big) \hat{\phi}\big(\mathcal{P}(\sigma)\big) | 0 \rangle, \end{aligned} \tag{3.18}$$

where we have used the symmetry of the system with respect to the stationary Killing field, and where we have set $\sigma = \tau/2$. Now consider the couple of events $(\mathcal{P}(-\sigma),\, \mathcal{P}(\sigma))$ on the worldline of the detector. From the point of view of the Märzke–Wheeler construction, these are the emission and detection events that exchange light signals with all the events of Märzke-Wheeler radius σ on the surface $\Sigma_{\bar{\tau}=0}$ of constant Märzke–Wheeler time $\bar{\tau} = 0$ (refer to Fig. 3.3).

Now, it is always possible to express the Klein-Gordon field $\hat{\phi}(x)$ by means of its propagator K. For instance,

$$\hat{\phi}\big(\mathcal{P}(\sigma)\big) = \int_S \mathrm{d}^2\mathcal{Q} \sqrt{{}^3g_S}\, \Big\{ K\big[\mathcal{P}(\sigma), \mathcal{Q}\big]\, \hat{\pi}(\mathcal{Q}) - K'\big[\mathcal{P}(\sigma), \mathcal{Q}\big]\, \hat{\phi}(\mathcal{Q}) \Big\}, \tag{3.19}$$

where S is the two-dimensional surface given by the intersection of the past lightcone of $\mathcal{P}(\sigma)$ with any *Cauchy surface*; in particular, we can take the intersection of the lightcone with the surface[14] $\Sigma_{\bar{\tau}=0}$. A similar construction is possible for $\mathcal{P}(-\sigma)$, so we get

$$R(\omega) = 2\int_{-\infty}^{+\infty} \mathrm{d}\tau\, e^{-2i\omega\sigma} F_{\bar{\tau}=0}(\sigma), \tag{3.20}$$

$$F_{\bar{\tau}=0}(\sigma) = \langle 0 | \left[\int_{\Sigma(\bar{\tau}=0,\sigma)} \mathrm{d}^2\Omega \sqrt{{}^3g_\Sigma} \left\{ K\hat{\pi}(\sigma,\Omega) - K'\hat{\phi}(\sigma,\Omega) \right\} \right]^2 | 0 \rangle. \tag{3.21}$$

Thus, the detector excitation probability is expressed by the Fourier transform of the expectation value $F_{\bar{\tau}=0}(\sigma)$ with respect to σ. The field operators that appear in $F_{\bar{\tau}=0}(\sigma)$ are evaluated on the constant-time surface $\bar{\tau} = 0$, and they are integrated over the Märkze-Wheeler angular coordinates[15] Ω.

In short, if we want to attribute the excitation of the accelerated detector to the presence of particles in the accelerated frame, then we can see that in some sense these particles reside on the surfaces of constant Märzke–Wheeler time. The *energy* of the particles (as seen by the accelerated detector) appears to be linked to some kind of radial frequency on the surfaces. Furthermore, because of the symmetry of the system, it does not matter which surface $\Sigma_{\tau'}$ we choose to compute $F(\sigma)$.

Our argument suggests that field quantization in Märzke-Wheeler coordinates might yield a notion of accelerated particle that is consistent with the measurements of accelerated detectors. Unfortunately, even writing (let alone solving) the Klein–Gordon wave equation in Märzke–Wheeler coordinates is fiendishly difficult.

Although the surfaces of constant Märzke–Wheeler time have a different shape from the constant–Lorentz-time planes implicit in rigidly rotating coordinates [Eq. (3.14)], the stationary Killing field used to translate the surfaces toward the future is the same. As we have seen, this Killing field becomes *spacelike* at large distances from the worldline. On the one hand, this circumstance raises some doubts about field quantization in Märzke–Wheeler coordinates, because the classical Cauchy problem for the wave equation is not well posed[16] when the time direction becomes spacelike (note however that field quantization in rigidly rotating coordinates suffers from the same problem).

On the other hand, Ashtekar and Magnon (1975) have shown that in stationary spacetimes,[17] field quantization is *unique* once we choose the

[14]The surfaces $\Sigma_{\bar{\tau}'}$ are not always Cauchy surfaces for the entire spacetime, but they always contain all the information necessary to reconstruct the value of the field at any point along the worldline $\mathcal{P}(\tau)$.

[15]At least for the Klein–Gordon field, the propagator K is a function of σ only.

[16]Actually, to our knowledge, it is unexplored.

[17]A spacetime is *stationary* if it is endowed with a timelike Killing field. Minkowski spacetime is stationary under any timelike combination of its ten Killing fields.

Killing field that provides the definition of positive and negative frequencies. If Ashtekar and Magnon's result applied to the rotating case, then Märzke–Wheeler quantization would be essentially equivalent to quantization in rigidly rotating coordinates, so nothing would be gained. However, Ashtekar and Magnon's theorem requires a temporal Killing field that is timelike everywhere. This leaves open the possibility that, for partially spacelike Killing fields, quantization might depend on the specific shape of the constant-time surfaces, so Märzke–Wheeler coordinates might provide a solution to the inconsistencies between the laboratory version and the intrinsic version of the Unruh effect.

Chapter 4

The Conventionality of Simultaneity

4.1 Conventionalism and geometry

At the beginning of the nineteenth century, the world of mathematics was shaken by a momentous discovery: Bolyai, Lobatschewsky (and possibly Gauss) built self-consistent geometries based on the negation of Euclid's axiom of the parallels, ending centuries of attempts to find a demonstration for the axiom. For centuries, Euclid's geometry had served as the model itself of mathematical knowledge. Euclidian geometry was a strong palace built on the self-evident truth of its foundations, the axioms; their strength was propagated upward to the theorems, through the transparently reliable process of logical inference. If Euclid's axioms were not logically necessary, according to Kant they could at least be regarded as inevitable.

Kant regarded space and time as the *a priori pure forms of intuition* of all sensible entities, and therefore as the preconditions for the validity of pure mathematics as a universal and necessary science; specifically, for the validity of Euclidian geometry and kinematics as the sciences of space and time. The actual structure and content of these disciplines could be investigated in terms of apodictic *a priori synthetic judgments*, precisely because they were *not* founded on the content of empirical acquaintance, but instead on the *a priori* universal pure intuitions. At the same time, this transcendental foundation accounted for the effectiveness of mathematics in the natural sciences (Kant, 1781; Friedman, 1992). For these reasons, the discovery of non–Euclidian geometries, rightly named the "Euclidian catastrophe" (Longo, 1999, 2001), "dissolved the idea itself of geometry as the univocal science of [physical] space, a fundamental idea that had always been implicitly incorporated in physics." (Pauri, 1997, p. 442)

Almost one century later, this philosophical tension was further increased by the advent of Einstein's theories of relativity. In his theory of special relativity (Einstein, 1905), Einstein showed by stringent reasoning that it was necessary to abandon the *absolute* physical time of Newton (which, just like Euclidian geometry, had been sanctioned by Kant as a descriptive necessity, in the sense of being the quantitative expression of an *a priori*, universal formal intuition) and replace it with a *relative* notion depending on the state of motion of the observer; furthermore, in his theory of general relativity (Einstein, 1915), Einstein accomplished an elegant description of gravitation that required the geometry of spacetime (and therefore of space) to be non–Euclidian.[1]

A number of thinkers [among them Riemann, Poincaré, Einstein himself, Reichenbach, Quine and Grünbaum (see Grunbaum, 1973)], set about to evaluate the philosophical and epistemological import of this revision in the scientific understanding of geometry and time. With some important distinctions, they converged on what came to be called the *conventionalist* position.[2] Just as Kant had suggested, they reasoned that physical theories must be dictated in part by *factual evidence*, and in part by *a priori*, formal and descriptive factors related to our conceptual organization of knowledge and experiment. However, the conventionalists held that these factors were by no means fixed: their choice was entirely *conventional*, suggested by criteria of convenience or developed by historical contingency.

4.1.1 The conventionality of geometry

Consider for instance the measurement of lengths, of primary concern in any discussion of geometry. A first convention, or *coordinative definition* (because it coordinates a theoretical construct to a physical object) lies in the choice of a physical unit of length, be it a metallic rod or the wavelength of a certain atomic excitation, and in the assumption that its *true* length *remains constant with time*. This assumption is not logically necessary, and there is no way to test it: with respect to what can we judge that *the unit of length itself* does not change with time? A second *coordinative definition* lies in the assumption that the unit of length remains unchanged when transported to another place; in other words, that measurements of length do not depend on the location at which they are performed. These considerations are not idle, because there could exist a *universal force* that affects all objects, including the basic physical unit, by changing their true length according to their location, or to time: in view of this, the notion of true length becomes elusive, whereas the coordinative definition of length with respect to the unit remains consistent (Reichenbach, 1928).

More to the point, given that different, unequivalent geometries are conceivable, any question about the actual geometry of space is a *physical ques-*

[1]The possibility of modeling dynamics using a curved geometry of space, and not of *spacetime*, had been explored earlier by Riemann (1867), without success.

[2]See (Norton, 1992) for a concise introduction.

tion that requires experiment: but any such question can only be asked for the joint construct of (coordinative definitions) + (universal forces). For example, consider a curved, non–free-mobility, non–Euclidian geometry, where all lengths are measured under the assumption that the unit of length is invariant by translation; in such a geometry, the circumference of circles is not in general 2π times their radius. Now consider a flat, Euclidian geometry where the true lengths are shrunk or enlarged at each location by an appropriate universal force, in such a way that circumferences are always measured to be 2π times their radius. These two geometries are indistinguishable *by any possible experiment*. Thus, neither of them can be held as more true, or more natural than the other. In Reichenbach's words (1928, p. 18), "the geometrical form of a body is no absolute datum of experience, but depends on a preceding coordinative definition."

Do not misunderstand this popular example of the interplay between coordinative definitions and universal forces: the latter are not extraneous objects that are needlessly tacked on to simpler descriptions that could stand on their own. It is especially clear that they are not if we extend our scope from geometry to physical theories, where the universal forces should be understood as implicit in the statement of the physical laws. A trivial example is the coordinative definition of the right-handedness of a system of axes, which can be chosen arbitrarily if the Lorentz force law (and whatever else) is changed accordingly. To summarize, according to the conventionalist strategy "theoretical schemes that are *prima facie* incompatible... are accepted as *interchangeable descriptions*, relatively to different *stipulations* about the coordinations which must be introduced, in the foundations of physical theories, between mathematical elements and specific empirical elements." (Pauri, 1997, p. 442)

Two remarks are appropriate here. First, coordinative definitions have to comply with a requirement of *consistency*: for example, the assumption that a certain unit of length is invariant with time might prove inconsistent if two such units, prepared in the same way and kept at the same spatial location, are seen to disagree after some lapse of time. The test of consistency is always an *experimental* matter.

Second: as a matter of practice, we cannot employ physical units uncritically, but we always need to make a series of corrections for the physical disturbances and the experimental imprecisions that limit the invariance of the units. That is, while we *declare* the unit of length to be the fundamental standard for our measurements, we recognize that we have to correct it for all the local perturbations (in Reichenbach's language, we can identify them because they act as *differential* forces, with different effects on different objects.)

The idealized *rigid bodies* (rods), employed by Einstein to *coordinate* the geometry of inertial frames, are especially vulnerable to differential disturbances: the physical realizations of rigid bodies are *solid* objects, held together by complicated internal forces, and susceptible to thermal expansion, gravity-gradient– and pressure–driven deformations, and so on. According to Reichenbach (1928), solid bodies become rigid in the limit in which the external (disturbing) forces are negligible with respect to the internal forces.

4.1.2 A critical viewpoint on conventionalism

More abstractly, physical units find their standard (and exacting!) idealization as *perfectly closed systems*; but this closure can only be evaluated in the general frame of the physical theory that we are building. This exception exemplifies one of the strongest arguments in the modern criticism of the conventional position: in a given physical domain, the distinction between the basic perceptive content (the object of coordinative definitions) and the theoretical constructs (which are induced from experiments and expressed in the language of the coordinative definitions) *depends on the physical theory as a whole*. Thus *truly incompatible* descriptive schemes are possible that disagree not only on the coordinative definitions made on the same physical entities, but also on which entities can be the objects of coordinative definitions (Pauri, 1997; Friedman, 1983).

This said, even for conventionalists not all coordinative definitions are truly equal. Although the sieve of truth cannot be used to sift them, they can be ranked according to the *descriptive simplicity* that they allow. To take one of Reichenbach's examples (Reichenbach, 1928), there is no point in replacing the description of a straight cord with that of a curved one, if a complicated universal force is then required to adjust the tension and support the curvature of the cord. The second description would be just as true as the first one, but much less useful. So even if all conventions are equally true, often there is a strong criterion to choose which convention to use in practice. This consideration is perhaps the most significant restriction on the epistemic import of conventionalism.

Nevertheless, the influence of conventionalism remains paramount for the whole field of the philosophy of spacetime: the conventionalist position succeeded in rejecting the Kantian claim that the structure of space and time was epistemically determined *a priori*; more important, it highlighted the status of space and time as legitimate (and inescapable!) objects of physical investigation, rather than implicit, unquestioned infrastructures of physical theories.

In the next sections, we shall examine closely the conventionalist claims about the notion of time; these claims have a special historical interest because they are prominent in Einstein's original exposition of special relativity. In particular, we shall be concerned with Reichenbach's discussion of the *conventionality of distant simultaneity*, and with his definition of *non-standard synchrony*.

4.2 Conventionalism and time

According to Reichenbach (1928), there are three basic coordinative definitions in the physical theory of time: the definition of the *unit of time*, the assumption of the *uniformity* of time, and the stipulation of a criterion for *distant simultaneity.*

The first two definitions are brought together by the definition of *clocks.* Whereas the *topological* nature of time is immediately comprehensible to our perception,[3] its metrical qualities are not so directly accessible. In fact, we never measure lapses of time, but only *processes.* Therefore, we base the measurement of time on assumptions regarding the behavior of certain physical systems, *clocks*, which we trust (or better, we *define*) *to embody the uniform unfolding of time.*

These systems fall in two classes: *closed periodic systems*, such as watches, atomic clocks, and the rotation of the Earth; and *clocks based on the measurement of distances*,[4] such as inertial clocks (clocks set in such a way that isolated bodies will move through equal distances in equal times), or clocks based on the movement of light. It is an experimental fact that both classes of clocks converge to the same, consistent definition of time.

Much like length, *uniform time* is obtained from *measured time* after a series of corrections, which take into account the imperfect closure of the clocks, and the known imperfections in their periodicity (for the first class of clocks) or uniformity (for the second class). Once again, these corrections are possible only within the general frame of a physical theory, so it seems untenable to follow Reichenbach in the distinction between *natural* clocks and clocks based on the laws of mechanics. The consistency of a convention for time can be tested only concurrently with the accuracy of the physical laws that use that convention;[5] the validation between clocks and laws is reciprocal, and indeed circular.

4.2.1 The conventionality of simultaneity

We come finally to the third coordinative definition in the physical theory of time: *simultaneity.* First, what do we mean by this notion? Aristotle formalized our intuitive appreciation of time when he described the *now*

[3]According to different philosophical positions, the topological structure of time can be identified either with the relation of temporal precedence of our experience, or with the relation of causal propagation (causal priority).

[4]The current metrological practice assumes the exact opposite, defining the unit of length from the unit of time, rather than the other way around. This is why the speed of light is now *defined*, not measured.

[5]This argument is not the same as the basic conventionalist claim that coordinative definitions are relative to the *universal* elements of physical laws: here the relevant physical laws have a *differential* effect on the internal workings of the clocks and ultimately on their supposed uniformity.

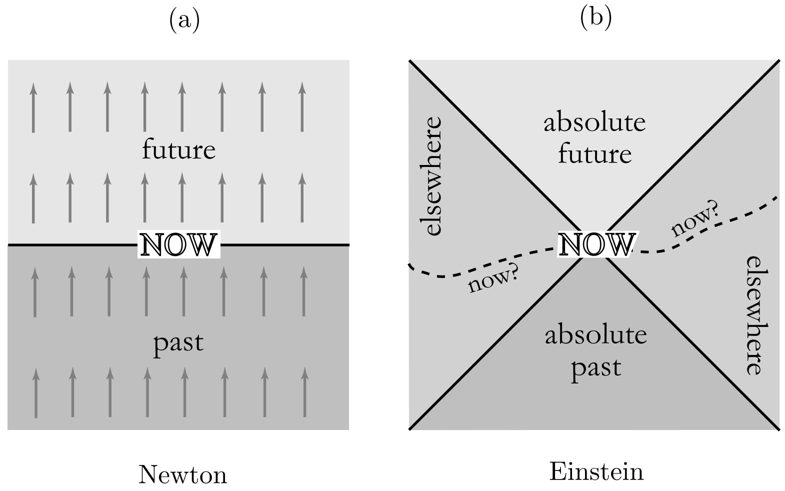

Figure 4.1: (a): Newtonian absolute simultaneity. (b): Relativistic causal structure.

Because in special relativity the speed of light sets a finite upper limit for the propagation of causal signals, for any event of spacetime there exists a four-dimensional spacetime region (the *elsewhere*) which is *causally disconnected.* Reichenbach and Grünbaum argue that the designation of a three-dimensional *simultaneity slice* through the elsewhere is a *conventional* operation. In this figure, time runs upward, space extends to the left and right.

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as the locus that separates what has been from what is yet to be. Two instants of time, experienced at distant locations (in short, two *events*), are simultaneous when they share (or have shared, or will share) just that special *now* (see Fig. 4.1a). Simultaneity is naturally conceived as a reflexive, symmetric and transitive relation that partitions the universe of events into classes; the events in each class share the same *past* and the same *future* (Torretti, 1996).

At this point, you might have already realized why special relativity jeopardizes this notion of simultaneity. We have just defined the *now* as the interface between *past* and *future.* In Newtonian physics (and in Kantian philosophy), past and future are obtained unequivocally from *causality.* If the event $\mathcal{A}$ *causes* the event $\mathcal{B}$ (even partially), but $\mathcal{B}$ does not cause[6] $\mathcal{A}$, then $\mathcal{A}$ precedes $\mathcal{B}$. In Newtonian theories where infinitely fast signals (and therefore, causal influences) are possible, this *law of causality* effectively establishes[7] simultaneity as an equivalence relation.

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[6]That is, any change of the state of the world at $\mathcal{A}$ would affect $\mathcal{B}$, but not *vice versa.*

[7]In fact, Kant identifies a second, direct criterion of simultaneity: $\mathcal{A}$ and $\mathcal{B}$ are simultaneous when they undergo *continuous interaction.* The typical example are Newtonian

In special relativity, the law of causality defines the *absolute past* and the *absolute future* of an event, but the union of these two regions does not cover spacetime completely. Because the speed of light sets a finite limiting velocity for the propagation of causal signals, for any event $\mathcal{A}$ there exists a *four-dimensional* spacetime region $\Sigma(\mathcal{A})$ which is *causally disconnected* from $\mathcal{A}$: that is, the events in S can neither *cause* $\mathcal{A}$ nor be caused by $\mathcal{A}$. According to Grünbaum (1973), the relation between $\mathcal{A}$ and the entire $\Sigma(\mathcal{A})$ (*topological simultaneity*) is the only *nonconventional* definition of simultaneity possible in special relativity. It is clear that topological simultaneity is not an equivalence relation; a coordinative definition is needed to cut a slice of *metrically simultaneous*[8] events through Σ (see Fig. 4.1b).

In operational terms, simultaneity is defined by statements about the relative rate of distant clocks, and about their *synchronization*. When we define clock time as *uniform*, we are making a statement about the comparison of time lapses measured at the same location, but at different times. In essence, because we have no way to check that today's clock minute is *congruent* to tomorrow's clock minute, we *define* them as congruent. Now, there are two related problems that force a *conventional definition of simultaneity*: first, there is no unique way to set the zero of distant clocks (that is, to say that my local clock's 8:00pm are truly simultaneous to your distant clock's 8:00pm); second, even when simultaneity is established for a couple of events, there is no way to compare the time lapses measured by distant clocks.[9]

Are these problems really unsolvable? Let us try this way: you and I synchronize our clocks while we are sitting together,[10] and after we satisfy ourselves that both our clocks run uniformly and at the same rate, you move away (see Fig. 4.2a). Then I can assume that the instants marked as 8:00pm, 8:01pm, ... on my clock are simultaneous to the instants marked by the same readings on your clock. Unfortunately, this is already a conventional definition (known as *synchronization by clock transport*), because I have no way of knowing how the rate of your clock might change (by universal forces) once you have left. Furthermore, we could verify that synchronization by clock transport is not consistent, because it is an empirical fact that moving clocks move slow with respect to stationary ones (Hafele and Keating, 1972a,b).

We then try to implement an altogether different strategy. While we are at rest at a distance, I send you a message (for instance, I raise a sign) asking you to send me your time. When your answer comes back, I use it to synchronize my clock with yours. The catch is that our signals travel with finite velocities, so I have to set my clock to the reading contained in

masses that are *acting at a distance* on one another.

[8]This is Grünbaum's expression.

[9]The problems are related because rate of distant clocks could be compared by repeated synchronizations.

[10]There is no convention in the synchronization of clocks *which share the same location.*

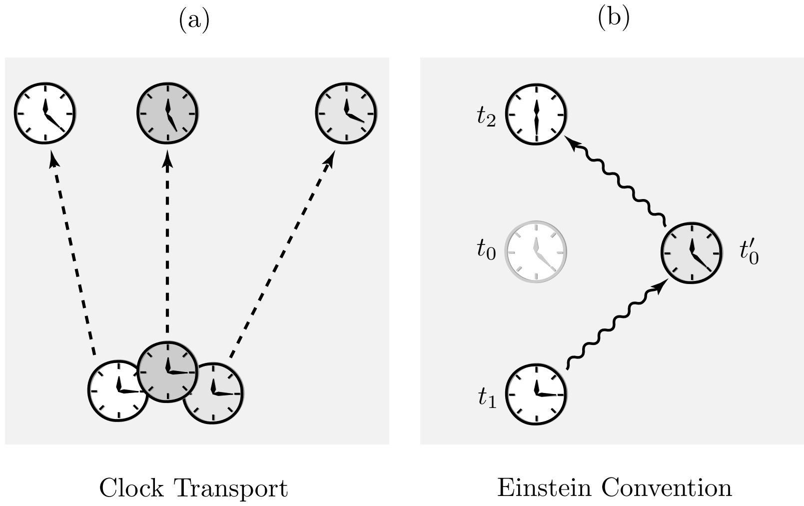

Figure 4.2: (a): Clock-transport synchronization. (b): Einstein synchronization.

Because in special relativity the relative rates of clocks depend on their velocity, clock-transport synchronization proves inconsistent. Einstein proposed a consistent synchronization procedure based on the exchange of light signals: the time t_2 on the white clock is chosen so that the white-clock event marked t_0, which lies halfway between the emission of the first signal and the reception of the second, is simultaneous with the gray-clock event t'_0 at which the first signal is received. In this figure, time runs upward, space extends to the left and right.

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your message, *plus* the time that your message took to reach me: that is, I can *infer* the time on your clock, *if I know* the velocity of our signals. *But to measure any velocity, we need first to secure two synchronized, distant clocks*, and that is just what we are trying to achieve! (We need one clock to measure the time at departure, the other to measure the time at arrival.) According to Reichenbach (1928), this circularity points to the need for a coordinative definition of distant simultaneity.

Everything would be fine if only we could exchange signals that travel with infinite velocity: in prerelativistic theories with infinitely fast signals it is possible (at least in principle) to achieve nonconventional synchronization simply by broadcasting instantaneous signals between distant clocks.[11] So the conventionality of simultaneity is a direct consequence of the empirical

[11] In fact, it is possible to employ the Einstein convention (with finite-velocity signals) in a Newtonian physics, although there is an element of perversity in doing so (see Redhead, 1993; Zahar, 1977). On the other hand, even in special relativity one could consider the use of superluminal signals, if they are available as tachyons, or as *superluminal phonons* (Redhead, 1993).

observation that there is a maximum speed for the propagation of causal signals.

4.2.2 Einstein synchronization

Einstein was very aware of the problem of simultaneity, which plays a central role in his seminal 1905 paper on special relativity. In this article, Einstein seeks to reconcile two apparently incompatible principles: the time-proven *principle of Galilean relativity*, by which the laws of physics cannot depend on the absolute velocity of the frame of reference,[12] and the *light principle* recently confirmed by Michelson and Morley, by which the speed of light *in vacuo* does not depend on the state of motion of the source. To accomplish this reconciliation, Einstein refuses to modify the equations of electromagnetism; instead, he embarks in a *conceptual criticism* of the notion of measurement (and therefore, of *reference frame*), whose final result are the Lorentz transformations.

A cornerstone of Einstein's criticism is the definition of a *global* time coordinate such that the description of free motion satisfies Newton's first law: that is, an *inertial* time coordinate. Einstein understands that once a fiducial clock is used to define time at a single spatial location, a convention is needed to propagate that time throughout space. He first examines a future-lightcone convention, by which the clocks (ideally) dispersed through space are synchronized to the readings that they receive from the fiducial clock by means of one-way light signals. With this convention, the surfaces of constant coordinate time are the future lightcones with vertices on the worldlines of the fiducial clock (see Fig. 4.3a; Fig. 4.3b shows a convention that uses *past* lightcones). Einstein rejects this time coordinate[13] because is not invariant with respect to the spatial translations of the fiducial clock. Furthermore, this time coordinate is not inertial! To see this, take an inertial particle whose worldline intersects the worldline of the fiducial clock (see Fig. 4.3c). As the particle passes by the clock, its velocity increases suddenly.

The article then goes on to introduce the *Einstein* (or *standard*) *synchronization convention.* To illustrate it, let us sit by two distant clocks at rest (see Fig. 4.2b). I note the time t_1 of my clock and send you a light signal; when you receive my signal, you immediately send back a light signal bearing the time t'_0 of your clock; I receive the second signal when my clock reads t_2. Einstein synchronization consists in setting my clock so that

$$t_0 = t_1 + \frac{t_2 - t_1}{2} = t'_0; \qquad (4.1)$$

[12] More elegantly: *no properties of phenomena correspond to the concept of absolute rest.*

[13] Probably Einstein had introduced it only as an example to show that there are other possibilities for the definition of time (Torretti, 1999).

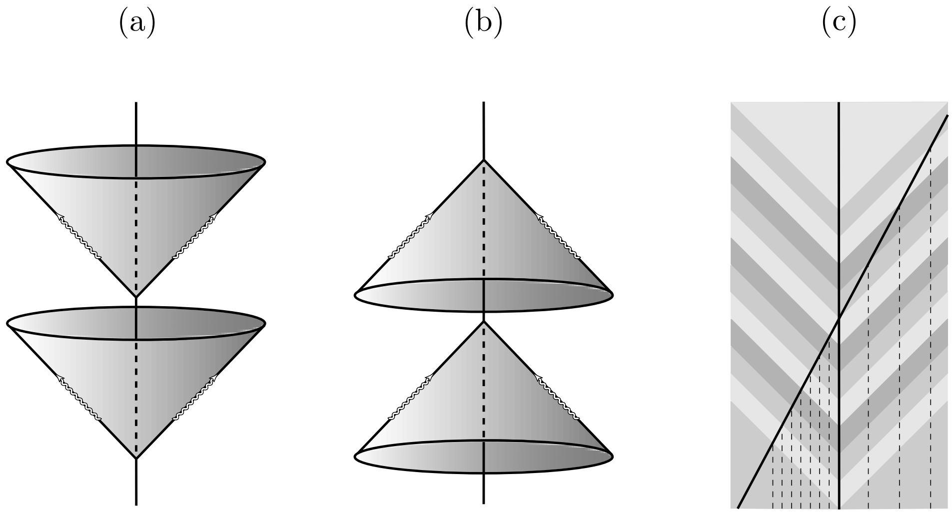

Figure 4.3: (a): Surfaces of simultaneity for the future-lightcone synchronization. (b): Surfaces of simultaneity for the past-lightcone synchronization. (c): Noninertiality of future-lightcone time.

In Fig. c, as an inertial particle (tilted line) passes by the fiducial clock (straight line), the velocity of the particle increases suddenly, as we can see by projecting the particle's worldline on the spatial axis at regular lightcone-time intervals. In this figure, time runs upward, space extends to the left and right.

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that is, we stipulate that the event at which you receive my signal is simultaneous to the particular event on my worldline that lies halfway between the emission of my signal and the reception of yours (see Figs. 4.2b, 4.4a). Loosely speaking, we stipulate that *you receive my signal halftime between the moment I emit it and the moment I receive your answer*. Let us repeat this once again, but in terms that make contact with our previous discussion of light-signal synchronizations (p. 47): Einstein synchronization is equivalent to the assumption that the magnitude of the one-way speed of the light signals in the two directions between the clocks is the same as the magnitude of the two-way speed of light in the round trip.[14]

4.2.3 Reichenbach's nonstandard synchronies

With his definition of inertial time and with the two fundamental principles that he endorses, Einstein is then able to derive the Lorentz transformations with all their consequences. But let us linger on Einstein's convention, and

[14] Remember that actually measuring the one-way speed requires two synchronized clocks!

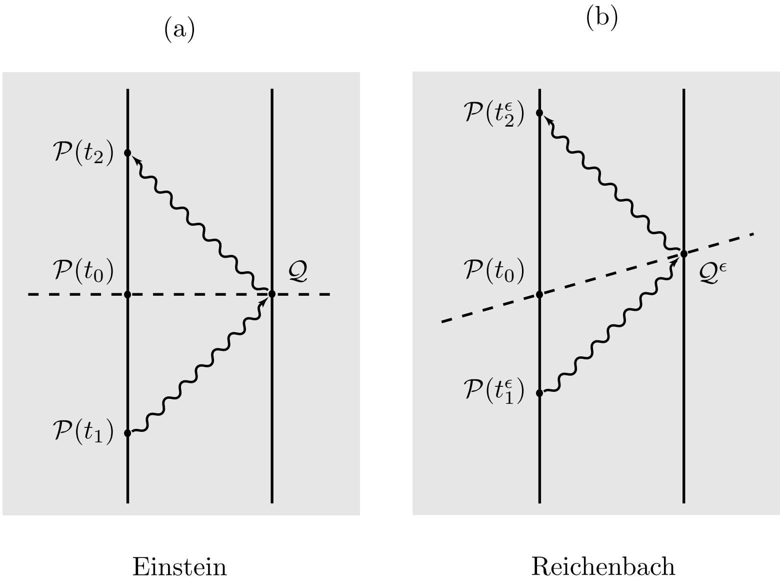

Figure 4.4: (a): Standard (Einstein) simultaneity. (b): Nonstandard (Reichenbach) simultaneity.

Both cases can be summed up by the formula $t_0 = t_1 + \epsilon(t_2 - t_1)$, where $\epsilon = 1/2$ for Einstein synchronization, $0 < \epsilon < 1$ for Reichenbach synchronization. For $\epsilon < 1/2$, $\mathcal{Q}^\epsilon$ moves to the future with respect to $\mathcal{Q}$. In this figure, time runs upward, space extends to the left and right.

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see how it could have been set differently. Reichenbach (1928) argues that instead of Eq. (4.1), we could have used any equation like

$$t_0 = t_1 + \epsilon(t_2 - t_1) = t_0', \quad \text{with } 0 < \epsilon < 1. \tag{4.2}$$

Note that if we took $\epsilon < 0$ or $\epsilon > 1$, we would locate the event at which you receive my signal respectively *before* I sent it, or *after* I received your answer (before or after in the *metrical* sense, of course). Reichenbach believed instead that when causal ordering is defined, metrical time ordering should agree with it. Apart from this, definition (4.2) is “adequate and could not be called false. If the special theory of relativity prefers the first definition [Einstein's], i. e., sets ϵ equal to 1/2, it does so on the grounds that this definition leads to simpler relations. It is clear that we are dealing here merely with descriptive simplicity.” (Reichenbach, 1928, pp. 127)

So, what does *Reichenbach* (or *nonstandard*) *simultaneity* look like? To see this, we shall start from Lorentz coordinates (whose constant–coordinate-time surfaces are implicitly Einstein simultaneous), and examine the equations for the surfaces of constant Reichenbach time. Remember, Einstein and Reichenbach synchronization are performed with respect to the inertial worldline $\mathcal{P}(t)$ of a fiducial clock, which we shall place at the Lorentz spatial coordinates $(x = 0, y = 0, z = 0)$; the coordinate time t will be the proper time of the clock. Now consider the worldline $\mathcal{Q}(t)$, which is at rest with respect to $\mathcal{P}(t)$, and which is described by $(x = x_\mathcal{Q}, y = y_\mathcal{Q}, z = z_\mathcal{Q})$. The event $\mathcal{Q}(0)$, which is Einstein simultaneous to $\mathcal{P}(0)$, has $t = 0$.

Einstein synchronization implies that there exist two light rays that connect $\mathcal{P}(t_1)$ to $\mathcal{Q}$ and $\mathcal{P}(t_2)$ to $\mathcal{Q}$, with

$$t_1 + (t_2 - t_1)/2 = 0, \tag{4.3}$$

$$(t_2 - t_1)/2 = |\mathcal{Q} - \mathcal{P}(0)| = \sqrt{x^2 + y^2 + z^2} = r. \tag{4.4}$$

The second condition follows from the description of lightcones in Lorentz coordinates, setting $c = 1$. Solving the two equations, we get $t_1 = -r$, $t_2 = r$. If we switch to Reichenbach's synchronization, Eq. (4.3) becomes$t_1 + \epsilon(t_2 - t_1)/2 = 0$; the solution is $t_1^\epsilon = -2\epsilon r$, $t_2^\epsilon = (1 - 2\epsilon)r$, and the event $\mathcal{P}(0)$ is Reichenbach simultaneous with

$$Q^\epsilon(0) : \big(t = (1 - 2\epsilon)r, x = x_\mathcal{Q}, y = y_\mathcal{Q}, z = z_\mathcal{Q}\big) \tag{4.5}$$

(see Fig. 4.4b). We can be general and let ϵ be a function of the spatial position (x, y, z) *and* the fiducial time (measured along $\mathcal{P}(t)$). Then the transformation between Lorentz time and Reichenbach time is

$$t^* = t + \big(1 - 2\epsilon(t, x, y, z)\big)r. \tag{4.6}$$

However, if ϵ is a function of time the actual implementation of the synchronization protocol requires some nonobvious bookkeeping! Furthermore, the constant-t^* slices are in general curved, undulating surfaces with non–Euclidian three-geometry (Fig. 4.5a). We can put some constraints on the function $\epsilon(t, x, y, z)$ by forbidding that the constant-t^* slices intersect, and by requiring that the spatial directions on the slices stay really spacelike. Then $\epsilon(t, x, y, z)$ must satisfy

$$\begin{gathered} \frac{\partial \epsilon(t, x, y, z)}{\partial t} < \frac{1}{2r}; \\ \Big|\big(2\epsilon(t, x, y, z) - 1\big)\frac{x^i}{r} + 2r\frac{\partial \epsilon(t, x, y, z)}{\partial x^i}\Big| < 1, \quad \text{with } (x^i = x, y, z). \end{gathered} \tag{4.7}$$

The simplest choice for ϵ, and indeed the choice originally suggested by Reichenbach (1928), is to take ϵ as constant and homogeneous. The resulting constant-t^* surfaces are hypercones with vertices along $\mathcal{P}(t)$ (Fig. 4.5b). The time t^*, however, *is not inertial* (unless of course $\epsilon = 1/2$, which corresponds to the Einstein convention and to Lorentz time). The velocity of a free particle passing through $\mathcal{P}(t_i)$ will be *discontinuous* at $t = t_i$, just as it happened for Einstein's future-lightcone convention (see Fig. 4.3c); indeed,

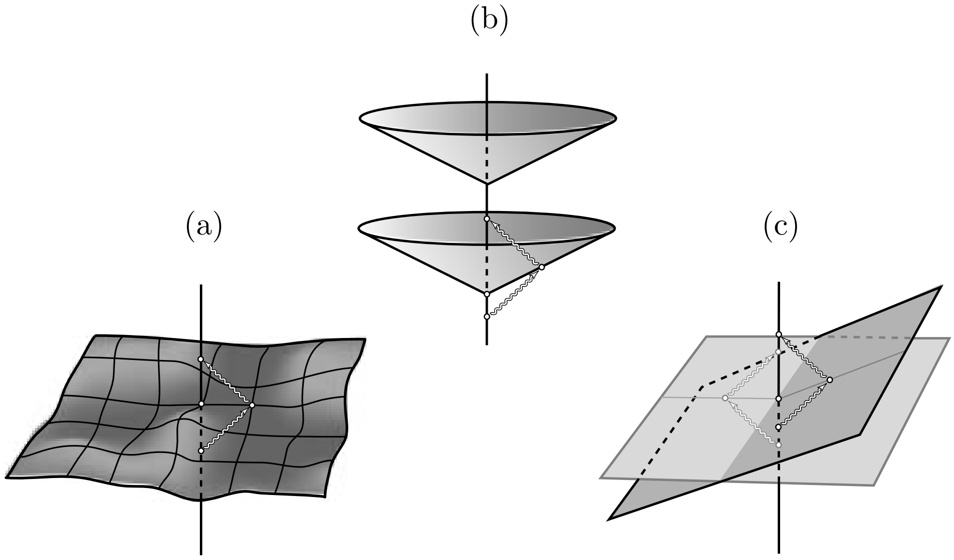

Figure 4.5: Reichenbach-simultaneous slices of spacetime.

(a) For a generic function $\epsilon(t, x, y, z)$, the surface $t^* = 0$ is curved.

(b) A constant and homogeneous ϵ yields a foliation of spacetime into hypercones (in this case, $0 < \epsilon < 1/2$).

(c) The requirement that the time coordinate be inertial yields *modified Reichenbach time* (4.8), which is simply the Lorentz coordinate time of an inertial worldline in motion with respect to our fiducial worldline.

In this figure, time runs upward, space extends to the left and right.

Reichenbach's synchronization reproduces the future-lightcone convention for $\epsilon = 1$.

Indeed, if viable synchronization procedures must yield *inertial* time coordinates [as in the spirit of Einstein's original discussion (1905)], then Reichenbach synchronization must be restricted to the form (Torretti, 1996)

$$t^* = t + \big(1 - 2\epsilon'\big)\hat{\mathbf{n}} \cdot \mathbf{r}, \tag{4.8}$$

where ϵ' is a constant, and $\hat{\mathbf{n}}$ is a fixed unit vector.[15] Eq. (4.8) expresses what Torretti (1999) calls *modified Reichenbach time*: that is, Reichenbach synchronization (4.2) with $\epsilon = \epsilon'$ for points that lie in the direction $\hat{\mathbf{n}}$ from $\mathcal{P}(t)$; with $\epsilon = 1 - \epsilon'$ for points that lie along $-\hat{\mathbf{n}}$; and with $\epsilon = 1/2$ (Einstein synchronization) for points that lie in the directions orthogonal to $\hat{\mathbf{n}}$. It turns

[15] Eq. (4.8) is a special case of Eq. (4.6): just take $\epsilon(x, y, z) = 1/2(1 - \epsilon''\hat{\mathbf{n}} \cdot \mathbf{r})$, with ϵ'' a constant.

out that if we adopt Eq. (4.8), then we are essentially using (except for a multiplicative constant) the Lorentz coordinate time of a frame that moves with velocity $(2\epsilon' - 1)\hat{\mathbf{n}}$ with respect to $\mathcal{P}(t)$ (Fig. 4.5c). In other words, in any given frame the only nonstandard synchronies that are compatible with the law of inertia coincide with the standard synchrony of other (boosted) frames. Torretti's trenchant comment is that "modified Reichenbach time is ... maladapted Einstein time" (1999, p. 277).

4.2.4 A physical critique of nonstandard synchrony

The strongest criticism against the conventionality of simultaneity is certainly the charge that Reichenbach and Grunbaum failed to realize how central Einstein synchronization is to the entire edifice of special relativity.

This centrality is evident from several points of view. First, we have seen that synchronization by clock transport proves to be inconsistent in special relativity, because the relative rates of clocks change according to their speed. However, synchronization by *infinitely slow* clock transport[16] is perfectly viable, as was first realized by Eddington (1924). It turns out that infinitely slow clock transport *is exactly equivalent to Einstein simultaneity* (Friedman, 1977, 1983; Torretti, 1996). The only assumption underlying the slow–clock-transport convention is that clocks measure the proper time along their worldlines:

$$d\tau = \sqrt{\eta_{ij} \frac{dx^i}{d\lambda} \frac{dx^j}{d\lambda}} d\lambda. \tag{4.9}$$

Thus, adopting nonstandard synchronism means having to change Eq. (4.9), "which is perhaps *the* central explanatory principle of special relativity." (Friedman, 1983, p. 317) Since Eq. (4.9) can be seen as the consequence of a convention, it cannot be tested directly; "but it is about as well confirmed as a theoretical principle can be." (Friedman, 1983, p. 317)

Second, standard synchronization is closely intertwined with Einstein's *light principle*, which sanctions the assumption that the one- and two-way speeds of light are the same, but which is true only under Einstein time. To establish his convention,

> Einstein picks a swarm of bouncing photons issuing at a particular instant from a point in an inertial frame. His definition of time implies that *these* photons move with the same speed in every direction. But once Einstein time is fixed in an inertial frame, *all other* photons bear witness to the validity of the light principle in that frame. (Torretti, 1999, p. 273)

[16] Defined as the limit of a sequence of synchronization procedures in which the clocks are moved from the origin with progressively slower speeds.

And, we add, they bear witness to the consistency of standard simultaneity. Thus, the exceedingly successful experimental verification of special relativity upholds the *combination* of Einstein synchronization and of the light principle. Give up one of them, and the other falls too.

Third, and most important, standard simultaneity is *explicitly definable* from the conformal structure of Minkowski spacetime, that is, from *causal* relations alone. This fact was implicit in the axiomatic systems given for Minkowski geometry by Robb (1914) and by Mehlberg (1935; 1937), and in Zeeman's *representation theorem* (Zeeman, 1964; Torretti, 1996), but it was finally brought to the attention of most philosophers of science by Malament (1977). We shall examine Malament's paper closely in Sec. 4.2.6, but let us first briefly comment on Zeeman's theorem.

Minkowski spacetime can be defined axiomatically as a *causal space*:[17] that is, for any two events of spacetime we can say whether one of them *causally precedes* the other, or whether they are *causally disconnected*. A *causal automorphism* is a mapping of a causal space into itself that preserves these causal relations. Zeeman's theorem states that the causal automorphisms of Minkowski spacetime are the Lorentz transformations and the dilatations. Under the assumption that Minkowski spacetime is flat, this result implies that the causal structure determines the metric up to a scaling factor. But we know that Einstein time is crucial to the definition of Lorentz frames (which are especially adapted to the Minkowski metric) and of Lorentz transformations (which are the symmetry group of Minkowski spacetime, and the *isometries* of the Minkowski metric): therefore, causality confers a very special status to standard simultaneity.

All these arguments strongly suggest that "standard simultaneity . . . is explicitly definable from the other quantities of relativity theory: it cannot be varied without completely abandoning the basic structure of the theory." (Friedman, 1983, p. 320) Not only the adoption of nonstandard synchrony casts a shadow on the explanatory power of some of the central tenets of special relativity, but it requires the addition of the ϵ field to Minkowski spacetime. This extra structure *serves no explanatory purpose.*

Consider for instance a reformulation of special relativity in terms of modified Reichenbach time (which at least salvages the *inertial* character of the time coordinate): Winnie (1970) proves that such a theory is *kinematically equivalent* to special relativity, in the sense that it makes the same *physical* predictions of special relativity in a different language (one with very punishing formulas). However, Winnie's work falls short of vindicating Reichenbach's claim that distant synchronization is conventional: as Friedman points out (1977; 1983), the *generally covariant* formulation of special relativity makes it possible to adopt *any* coordinate system (and not just modified Reichenbach systems), and still live up to kinematical equiva-

[17] See Kronheimer and Penrose (1967); for a simple exposition, see Torretti (1996).

lence. Indeed, "if Winnie's equivalence claim is to have any real content ... it should say something about the intrinsic spacetime structures described by our theory, not just about the different coordinate systems in which these structures are represented." (Friedman, 1983, p. 175)

Instead, nonstandard-synchrony reformulations of special relativity serve the only purpose of cloaking the physical content of the theory under a layer of unnecessary descriptive complication; furthermore, this layer depends completely on the arbitrary (and unphysical!) specification of ϵ throughout spacetime.

4.2.5 A philosophical critique of nonstandard synchrony

To these arguments Reichenbach might of course answer that the advantage of standard synchrony resides only in the greater *descriptive simplicity* that it affords, and in nothing else. We can raise at least two objections to this defense. According to Friedman,

> Reichenbach argues from an epistemological point of view; he argues that certain statements are conventional as opposed to "factual" because they are unverifiable in principle. ... the main problem with Reichenbach's argument is this: whether or not statements about distant simultaneity are in some sense unverifiable in the context of special relativity, we have been given no reason to suppose that unverifiability implies lack of determinate truth value. (Friedman, 1977, pp. 426–428)

In other words, we are entitled to claim that standard synchrony yields a truthful description of physical time, even if we have to recognize that simultaneity is not directly verifiable. Furthermore, as underlined by Norton, conventionalists must try to avoid the pitfall of an

> indiscriminate antirealism in which any law is judged conventional if the law fails to entail observational consequences without the assistance of other supplementary laws. ... The Duhem–Quine thesis ... states that it is impossible to test the individual laws of a theory against experience. We can only test the entire theory. Any attempt to test individual laws will fail since we can always preserve any nominated law from falsification by modifying the other laws with which it is conjoined when we derive observational consequences from it. (Norton, 1992, p. 189)

To this point, most conventionalists would counter that we can speak meaningfully of conventionality only in those cases where

> conventionality depends on a very small vicious circle that must be broken by definition since no independent factual test is pos-

> sible for the individual components of the theory. The Duhem–Quine thesis does not restrict the manner in which we might protect a law from falsification. We might have to do so by a complicated and contrived set of modifications spread throughout the theory. Some of the components modified may be subject to independent test and thus not properly susceptible to conventional stipulation. (Norton, 1992, p. 189)

We believe that the physical arguments presented in the last section are sufficient to establish that the vicious circle involved in the definition of distant simultaneity is not small, but it encompasses almost all of special relativity.

Grünbaum's position (1973) appears even weaker. As we have briefly seen in Sec. 4.2.1, Grünbaum does not take issue with the verifiability of what he calls *metrical* simultaneity; rather, he maintains that the only nonconventional notion of simultaneity is *topological* simultaneity, which can be defined directly from causal relations (whereas *metrical* simultaneity cannot be so defined). Grünbaum's conviction fits his version of the *causal theory of time*,[18] where the only *objective* temporal relations are the causal relations between events. This ontological position can be criticized easily: it is not at all clear that the list of the *objective* properties of the physical world can be drawn *a priori*, rather than being compiled on the basis of the best available physical theories (Friedman, 1983). Whereas Grünbaum's objections of principle are suspicious, he is plainly wrong on matters of fact: as we have seen, Einstein simultaneity *is* definable entirely from causal relations. In fact, it is the *only* nontrivial simultaneity relation that is so definable, and so it must be considered nonconventional by right. This is the upshot of Malament's 1977 theorem (Malament, 1977), which is the subject of the next section.

4.2.6 Malament's argument: simultaneity from causality

In his 1977 paper, Malament expands some arguments due to Robb (1914), proving how standard simultaneity is essentially *orthogonality in Minkowski spacetime*, and how that orthogonality can be defined from causal relations alone. To show this we need first to establish a connection between causal relations and the affine structure of Minkowski spacetime. It is a basic assumption of special relativity that the causal structure of Minkowski spacetime is encoded in the *Minkowski inner product* of two events,

$$(\mathcal{P}, \mathcal{Q}) = -t[\mathcal{P}]t[\mathcal{Q}] + x^1[\mathcal{P}]x^1[\mathcal{Q}] + x^2[\mathcal{P}]x^2[\mathcal{Q}] + x^3[\mathcal{P}]x^3[\mathcal{Q}] \quad (4.10)$$

(although this expression makes recourse to Lorentz coordinates, it is of course Lorentz invariant). The three symmetric relations of *causal con-*

[18] For the causal theory of time, see (van Fraassen, 1985).

nectibility (κ), *timelike relatedness* (θ), and *lightlike relatedness* (λ) are then defined by

$$\begin{aligned} \mathcal{P}\kappa\mathcal{Q} \quad &\text{iff} \quad |\mathcal{P}-\mathcal{Q}| = (\mathcal{P}-\mathcal{Q},\mathcal{P}-\mathcal{Q}) \leq 0, \\ \mathcal{P}\lambda\mathcal{Q} \quad &\text{iff} \quad |\mathcal{P}-\mathcal{Q}| = (\mathcal{P}-\mathcal{Q},\mathcal{P}-\mathcal{Q}) = 0, \\ \mathcal{P}\theta\mathcal{Q} \quad &\text{iff} \quad \mathcal{P}\kappa\mathcal{Q} \;\&\; \neg\mathcal{P}\lambda\mathcal{Q}; \end{aligned} \tag{4.11}$$

with some cleverness, it is actually possible to use any one of κ, θ, and λ to define the other two (Malament, 1977, Note 3), and even to reconstruct the full inner product (Robb, 1914). Now, standard simultaneity with respect to the inertial worldline $\mathcal{P}(\tau)$ is defined as follows: *$\mathcal{Q}$ is Einstein simultaneous to $\mathcal{P}_0 \in \mathcal{P}(\tau)$ iff there exist two other events $\mathcal{P}_1$, $\mathcal{P}_2$ on $\mathcal{P}(\tau)$ such that*

$$(\mathcal{P}_0-\mathcal{Q},\mathcal{P}_2-\mathcal{P}_1) = 0. \tag{4.12}$$

Since the Minkowski inner product can be defined from κ, so can standard simultaneity. To see that the relation implied by Eq. (4.12) is really standard simultaneity, let us rephrase Eq. (4.2) as

$$\mathcal{P}_0 = \mathcal{P}_1 + \epsilon(\mathcal{P}_2-\mathcal{P}_1); \tag{4.13}$$

we insert this definition into Eq. (4.12):

$$\begin{aligned} 0 =&(\mathcal{P}_0-\mathcal{Q},\mathcal{P}_2-\mathcal{P}_1) = \\ =&\big(\mathcal{P}_1+\epsilon(\mathcal{P}_2-\mathcal{P}_1)-\mathcal{Q},\mathcal{P}_2-\mathcal{P}_1\big) = \\ =&\epsilon|\mathcal{P}_2-\mathcal{P}_1| + (\mathcal{P}_1-\mathcal{Q},\mathcal{P}_2-\mathcal{P}_1) = 0; \end{aligned} \tag{4.14}$$

by our operational definition of synchronization, we have $\mathcal{P}_1\lambda\mathcal{Q}$ and $\mathcal{P}_2\lambda\mathcal{Q}$; hence,

$$\begin{aligned} 0 =&|\mathcal{P}_2-\mathcal{Q}| = |\mathcal{P}_2-\mathcal{P}_1+\mathcal{P}_1-\mathcal{Q}| = \\ =&|\mathcal{P}_2-\mathcal{P}_1| + 2(\mathcal{P}_1-\mathcal{Q},\mathcal{P}_2-\mathcal{P}_1) + |\mathcal{P}_1-\mathcal{Q}| = \\ =&|\mathcal{P}_2-\mathcal{P}_1| + 2(\mathcal{P}_1-\mathcal{Q},\mathcal{P}_2-\mathcal{P}_1) = 0. \end{aligned} \tag{4.15}$$

Combining the last lines of Eqs. (4.14) and (4.15), we get $(\epsilon-1/2)|\mathcal{P}_2-\mathcal{P}_1| = 0$, which implies that $\epsilon = 1/2$, as appropriate for the standard synchronization convention.

Malament is then able to prove that standard synchronization is the only nontrivial simultaneity relation that can be defined from κ and $\mathcal{P}(\tau)$. He does so by imposing three conditions on any candidate simultaneity relation S:

1. S must be an equivalence relation;
2. S must relate events on $\mathcal{P}(\tau)$ to events not on $\mathcal{P}(\tau)$, but S cannot be a *universal* relation that relates every event of spacetime to every other event (nontriviality);

3. S must be preserved under the $\mathcal{P}(\tau)$ *causal automorphisms* (the mappings of Minkowski spacetime onto itself that also map the worldline $\mathcal{P}(\tau)$ onto itself).

The third condition is crucial, and it seems a necessary consequence of defining S from κ and $\mathcal{P}(\tau)$ *alone*: any transformation that leaves κ and $\mathcal{P}(\tau)$ unaltered should also map simultaneous points into simultaneous points.

We will give only a brief sketch of Malament's proof.[19] The $\mathcal{P}(\tau)$ causal automorphisms include all the rotations, translations, scalar expansions and reflections which map $\mathcal{P}(\tau)$ onto itself. Consider two events $\mathcal{P}_0 \in \mathcal{P}(\tau)$ and $\mathcal{Q} \notin \mathcal{P}(\tau)$ which are simultaneous under S [briefly, $S(\mathcal{P}_0, \mathcal{Q})$]. Let $\Pi_{\mathcal{P}_0}$ be the set of the $\mathcal{P}(\tau)$ causal automorphisms that have $\mathcal{P}_0$ as a fixed point; now let $\Pi_{\mathcal{P}_0}(\mathcal{Q})$ be the set of all the events obtained by applying the elements of $\Pi_{\mathcal{P}_0}$ to $\mathcal{Q}$. The events in $\Pi_{\mathcal{P}_0}(\mathcal{Q})$ are all S simultaneous with $\mathcal{P}_0$ and with each other. If $\mathcal{P}_0$ and $\mathcal{Q}$ are simultaneous under the standard simultaneity relation S_0, then $\Pi_{\mathcal{P}_0}(\mathcal{Q})$ is just the hypersurface orthogonal to $\mathcal{P}(\tau)$ that contains $\mathcal{P}_0$. If they are not, then $\Pi_{\mathcal{P}_0}(\mathcal{Q})$ is a *double cone* (but not necessarily a lightcone!) with vertex $\mathcal{P}_0$. In this case, we can prove that $S(\mathcal{P}_0, \mathcal{R})$ *for all the events* $\mathcal{R}$ *of spacetime*, so S must be universal. Let us see how.

Suppose first that $\mathcal{R} \in \mathcal{P}(\tau)$. There is a $\mathcal{P}(\tau)$ causal automorphism f which maps $\mathcal{P}_0$ onto $\mathcal{R}$, and $\mathcal{Q}$ into $f(\mathcal{Q})$. The catch is that the double cone $\Pi_{\mathcal{R}}(f(\mathcal{Q}))$ must intersect the double cone $\Pi_{\mathcal{P}_0}(\mathcal{Q})$ at some event $\mathcal{S}$; because S is an equivalence relation, $S(\mathcal{P}_0, \mathcal{S})$ and $S(\mathcal{R}, \mathcal{S})$ imply $S(\mathcal{P}_0, \mathcal{R})$. Suppose now that $\mathcal{R} \notin \mathcal{P}(\tau)$. Then there is a $\mathcal{P}(\tau)$ causal automorphism g which maps $\mathcal{Q}$ onto $\mathcal{R}$. Since g preserves S, we have $S(g(\mathcal{P}_0), g(\mathcal{Q}))$, or $S(g(\mathcal{P}_0), \mathcal{R})$. But $g(\mathcal{P}_0) \in \mathcal{P}(\tau)$, so the first part of this paragraph can be used to prove that $S(\mathcal{P}_0, g(\mathcal{P}_0))$; therefore $S(\mathcal{P}_0, R)$. The idea is that if S shares a couple of simultaneous events with S_0, then it *contains* S_0; but if S has a couple of simultaneous events that are not in S_0, then S must be the universal relation. Ultimately, S can only be empty, universal, or S_0. If S is nontrivial, then S is S_0.

Sarkar and Stachel (1999) criticize Malament's proof because it includes a crucial assumption "that is physically unwarranted: any simultaneity relation must be invariant under temporal reflections." (Sarkar and Stachel, 1999) If this assumption is dropped, both the future-lightcone convention[20] and the specular past–lightcone convention become as viable as standard synchronization. Already in 1981, Spirtes had realized that Malament's result was vulnerable to losing even one of the requirements of invariance. However, why drop temporal reflections? Sarkar and Stachel's argument comes down to the claim that only the proper orthochronous subgroup of

[19] Other than Malament (1977), see (Norton, 1992; Anderson et al., 1998); for a beautiful geometric characterization, see (Redhead, 1993).

[20] As we already wrote (Sec. 4.2.2) Einstein did consider briefly this convention in his 1905 article.

Lorentz transformations can be realized physically.

Rynasiewicz (2000) rejects Sarkar and Stachel's claims very firmly. First of all, temporal reflections cannot be rejected for physical reasons, because under any accepted sense of definability S must respect *all* the symmetries shared by the entities from which it is defined; in this case, S must respect all the $\mathcal{P}(\tau)$ causal automorphisms. "What relations are definable from what is not a matter of physical adequacy, but rather is a purely formal, mathematical question." (Rynasiewicz, 2000) Second, Rynasiewicz is able to show that Sarkar and Stachel's reasoning is flawed when they purport to prove that the future and past lightcones emanating from an event are definable from causal relations alone; they are not, although for very technical reasons (Rynasiewicz, 2000).

4.2.7 A final word on the conventionality of simultaneity in special relativity

According to other authors, Malament's theorem is correct, but it falls short of proving that standard simultaneity is nonconventional (Redhead, 1993; Anderson et al., 1998). For instance, we can circumvent the theorem by accepting notions of simultaneity that are not relations of equivalence (Redhead, 1993). Alternatively, we can realize that in a given inertial frame, we are free to use the standard synchrony defined in *all other inertial frames* (Janis, 1983; Debs and Redhead, 1996); from the point of view of Malament's theorem, this would mean that some additional structure other than λ and $\mathcal{P}(\tau)$ (another worldline $\mathcal{P}'(\tau)$) is used to define S.

In this author's opinion, Malament's result seals conclusively the arguments outlined in Secs. 4.2.4 and 4.2.5, which suggest that *within the framework of special relativity*, standard simultaneity should be regarded as nonconventional because it is intimately tied to the physical content of the theory; indeed, adopting nonstandard simultaneity seems to contradict that physical content, or at least to obscure it beyond a layer of *arbitrary* descriptive complication. True, we must impose some requirements on simultaneity before we can regard it as truly nonconventional: for instance, the time coordinate must be inertial, and simultaneity must be an equivalence relation; but these requirements *serve a precise and fundamental purpose within special relativity.* Indeed, if we lift these constrictions, we gain much latitude in the definition of synchrony, but we do not enlarge the scope of special relativity, or provide any further physical insight—if anything, we get less! Ultimately, as Rynasiewicz correctly notes,

> neither Reichenbach nor Einstein would have accepted [the claim that temporal relations are nonconventional if they are definable uniquely in terms of causal connectability]. The question for them was not what we are at liberty to say about distant simul-

taneity after the theory of relativity is given, but rather to what degree one is constrained prior to the articulation of the theory. (Rynasiewicz, 2000)

Nevertheless, the whole debate on these issue has never strayed very far from the bounds of special relativity! Maybe Rynasiewicz is right when he writes that because of this, "most of the discussions [of nonstandard synchrony] simply miss the mark" (Rynasiewicz, 2000); but we could not agree more with Torretti when he states: "Restriction of the [simultaneity] rule to special relativity inevitably curtails the philosophical significance of Reichenbach's doctrine; but without this restriction it is far too indefinite to be argued for or against."(Torretti, 1996, p. 223)

4.3 Einstein's synchronization beyond inertial observers

In Ch. 3 we discussed the extension of the Einstein synchronization convention to noninertial observers, and we used the extended convention to build a system of accelerated coordinates (*Märzke–Wheeler coordinates*) adapted to the motion of a generic accelerated observer in Minkowski spacetime. Märzke–Wheeler coordinates display the following desirable properties: (a) on the worldline of the fiducial observer, the Märzke–Wheeler time coordinate coincides with the observer's proper time; (b) furthermore, in a neighborhood of the worldline, Märzke–Wheeler coordinates reduce to Lorentz coordinates; (c) the procedure assigns smoothly and unambiguously a Märzke–Wheeler time and a Märzke–Wheeler radial coordinate to all the events that lie in the *causal envelope* of the worldline.[21] Most important, Märzke–Wheeler time indexes a smooth foliation of the causal envelope into spacelike *surfaces of simultaneity*.

In the context of this chapter, one question then becomes natural: to what extent is the choice of Märzke–Wheeler time for accelerated observers *conventional*? To provide an answer, we will first have to make some distinctions between the notions of *coordinate set*, *reference frame*, and *observer*, which we have used somewhat loosely in the rest of this work. We will then move on to examine which *factual* and which *conventional* elements are required by the Märzke–Wheeler procedure.

4.3.1 Coordinate systems, reference frames, and observers

In the standard expositions of special relativity it is customary to blur the distinction between the notion of *coordinate system* and that of *frame of*

[21] In Sec. 3.3 we defined the *causal envelope* of a worldline as the intersection of its causal past and its causal future. The causal envelope contains all the events from which bidirectional communication with the worldline is possible.

reference. "The first ... is understood simply as the smooth, invertible assignment of four numbers to events in spacetime neighborhoods. The second ... refers to an idealized physical system used to assign such numbers." (Norton, 1993)

In special relativity, a reference frame is traditionally conceived as a regular lattice of identical rigid rods, at rest relatively to each other, plus a set of ideal clocks; the lattice and the clocks are distributed homogeneously throughout space. Using Reichenbach's terminology (Reichenbach, 1928), we may say that the special-relativistic notions of distance and time are *coordinated* to the rods and clocks,[22] so we read the clocks to know the time and we count the rods to measure distances. Lorentz coordinates are *adapted* to the standard reference frames of special relativity, so they have an immediate *metrical significance*: they are not simply arbitrary event labels, but they give measures of length and time according to the coordinative definitions adopted for the rods and clocks.

Two remarks are appropriate here. First, a definite state of (inertial) motion is associated to each reference frame; as a matter of fact, whenever we say that some notion is *relative* in special relativity, we mean it with respect to reference frames in different states of inertial motion. As a set of *active* transformations, the Lorentz group modifies the state of motion of frames; as a set of *passive* transformations, it provides mappings between the Lorentz coordinate systems adapted to frames in different states of motion.

Second, because of the homogeneity of Minkowski spacetime, we only need *one* inertial worldline to specify a reference frame: for instance, we can take the trajectory of the origin of the frame. We obtain the trajectory of all the other lattice points by parallel transport of this fiducial worldline. This circumstance has encouraged the *identification* of the notion of reference frame with that of *inertial observer* (who lives on a single worldline). It follows that the equations of physics, as expressed in a Lorentz coordinates, are naturally understood as referring to the measurements made by a single inertial observer.

This is not strictly true, because obviously an observer can measure quantities only along her own worldline. Yet the attribution of a global coordinate system to a local observer still seems very reasonable. In Sec. 3.1, we remarked that for inertial observers, Lorentz coordinates are a device to extend the concept of physical reality from the worldline to the entire spacetime, building a description of the world which incorporates notions of *distance*, and *simultaneity*. To extend her physical reality, the observer exploits the homogeneity of Minkowski spacetime, envisaging a homogeneous, space-filling population of her clones, each one of them armed with a clock. After all the clocks are synchronized, the clones begin making measurements

[22] Whether this coordination has a conventional character is what we have been discussing for the last 20 pages.

and exchanging data. The resulting physical picture obeys the equations of physics as written in the Lorentz coordinate system adapted to the original frame (and therefore to the original observer).

As soon as we abandon inertial reference frames, this tight integration among frames, coordinates, and observers begins to break apart (Norton, 1993). With the introduction of uniformly accelerated frames, coordinate time ceases to correspond directly to the measurements of clocks; with uniformly rotating frames, spatial geometry becomes non–Euclidian, so the direct coordination of spatial coordinates to rigid rods breaks down. Going on to curved spacetimes, it becomes impossible to specify coordinate systems that have metrical value everywhere; moreover, our imagination would be severely strained by the attempt to devise a global reference frame based on some generalization of a lattice.[23]

It seems that the years have brought consensus (Norton, 1993) around the definition of a *general-relativistic reference frame* as a spacetime-filling *congruence of curves*,[24] which are typically timelike geodesics. We can imagine that one observer lives on each such curve, carrying a clock and maybe a gyroscope (mathematically, this corresponds to specifying a tetrad field over spacetime; one vector of the tetrad points along the proper time direction). We can then define an adapted coordinate system (*Fermi coordinates*) by setting coordinate time to proper time, and fixing each of the curves of the congruence to constant spatial coordinates. In any case, the choice of a single geodesic observer is now insufficient to specify a reference frame. Indeed, we need a *population* of observers who cannot be cloned from each other by parallel transport.[25]

4.3.2 Elements in the definition of Märzke–Wheeler simultaneity

Let us go back to the Märzke–Wheeler construction of Ch. 3. Our purpose there was to define an accelerated *coordinate system* adapted to the motion of a generic accelerated observer in Minkowski spacetime. Mind you, a coordinate system, not a reference frame! For how could we devise a frame that is adapted to a *single* noninertial observer? We could try to link the observer to some kind of rigid, space-filling structure: but in special relativity it is impossible to accelerate extended rigid bodies without

[23] At the very least, it will become necessary to support the lattice at various points (for instance with rockets), and to counteract the stresses induced by curvature. Of course these tricks negate the spirit itself of setting up reference frames, which is to identify simple, reliable physical objects or procedures that can be safely coordinated to theoretical constructs.

[24] A set of curves that fill up spacetime without intersecting.

[25] At least in general. Sometimes curved spacetimes have enough symmetry to suggest a geometrically obvious (and in some sense, homogeneous) family of curves. One example are Robertson–Walker spacetimes, studied in cosmology (Torretti, 1996).

deforming their structure (Rindler, 1977). Alternatively, we could imagine a space-filling population of clones of the fiducial observer: but we have no unique prescription on how to replicate the original worldline throughout spacetime. A possible geometrical prescription could use the symmetries of Minkowski spacetime to suggest the geometry of replication; but it would fail to produce a frame adapted to the accelerated worldline, which does not share those symmetries.

Instead, we can define Märzke–Wheeler coordinates entirely from operations that the accelerated observer can carry out on her own worldline. Admittedly, she still needs the collaboration of other observers (or devices) distributed throughout spacetime, to send her the results of their measurements and to complete the Einstein triangulation of light signals: but she does not need to know anything about the *trajectory* of those other observers. So the only elements that really come into the definition of Märzke–Wheeler simultaneity are the accelerated worldline, the causal structure of Minkowski spacetime (implicitly, through the exchange of light signals), and *one* ideal clock that marks *proper time* along the worldline.

It seems that the situation here is not much different from what it was for the standard synchronization of inertial observers. Before concluding that Märzke–Wheeler simultaneity is also nonconventional, however, we should note that Märzke–Wheeler time *is not inertial*: as the curved geometry of the Märzke–Wheeler simultaneity slices might already suggest, inertial bodies will not move through equal distances in equal times (in fact, their trajectories will not even be straight lines in Märzke–Wheeler coordinates!) We should not be surprised that this is the case: from Newtonian physics, we are all used to the *apparent forces* that come into the equations of physics when these are written in accelerated frames. Much in the same way, the description of physics in the accelerated Märzke–Wheeler coordinates is implicitly a function of the acceleration of the worldline, which enters the description as an *absolute object* in the sense of Anderson (1967). "Roughly speaking, an absolute object affects the behaviour of other objects but is not affected by these objects in turn." (Anderson, 1969, p. 1657).

4.3.3 An aside on the notion of absolute objects

It is worth to spend a few lines to discuss Anderson's absolute objects (Anderson, 1967), just because they are a very interesting notion on their own. The Minkowski metric of special relativity is an absolute object because it affects the behaviour of other physical objects, the dynamical objects of the theory (for instance, it determines the trajectories of free bodies); but it is not affected by the dynamical objects in the way that, for instance, the general relativistic metric is: as a matter of fact, Anderson would say that general relativity is a theory *without* absolute objects.

Absolute objects are the basis for Anderson's distinction between the *covariance group* of a theory and its *symmetry* or *invariance group.* The dynamical objects of the theory transform under the covariance group; the absolute objects also transform under the covariance group, assuming different forms as they do so, but they are always essentially the same. *The symmetry group is the subgroup of the covariance group that leaves invariant the absolute objects.*

Let us take special-relativistic field theory as an example: the field ϕ^μ transforms under the Poincaré group as a four-vector; under the same group, the Minkowski metric $\eta_{\mu\nu}$ transforms trivially like a two-tensor, because it is invariant. So the Poincaré group is *both* the covariance group and the symmetry group of the theory. Now take the same special-relativistic classical field theory, in a generally covariant form where general coordinate transformations are allowed. Here ϕ^μ transforms like a vector, and $\eta_{\mu\nu}$ really transforms like a two-tensor, in the sense that under appropriate transformations, it might become nondiagonal, its determinant might change, and so on. Nevertheless, $\eta_{\mu\nu}$ is still invariant under Poincaré transformations. So the general group of diffeomorphisms is the covariance group of the theory, but the Poincaré group is still the symmetry group.

Because there are no absolute objects in general relativity, its symmetry group coincides with its covariance group, the general group of diffeomorphisms. In this admittedly formal sense, general relativity succeeds in generalizing the relativity of the special theory from the inertial coordinate systems related by Lorentz transformations to the general coordinate systems related by generic diffeomorphisms.

But let us go back to Märzke–Wheeler simultaneity. For any physical theory expressed in Märzke–Wheeler coordinates, the accelerated trajectory of the fiducial observer will be an absolute object, *on a par* with the Minkowski metric, because together they determine the 3-metric on the Märzke–Wheeler simultaneity slices. One could imagine a covariant version of this accelerated theory where the basic Märzke–Wheeler coordinates are transformed to some functions of themselves, while the 3-metric transforms accordingly. However, because a generic accelerated worldline shares no symmetry with the Minkowski metric, the symmetry group of the covariant accelerated theory will contain only the identity.

4.3.4 The conventionality of Märzke–Wheeler simultaneity

What about the conventionality of Märzke–Wheeler simultaneity, then? We have shown how this synchrony can be defined from a minimal set of elements (causal structure, the accelerated worldline, proper time), and we have come to regard the noninertiality of Märzke–Wheeler time as inevitable. Now consider a Reichenbach–Märzke–Wheeler synchrony where the time coordinate is defined as $\bar{\tau}^\epsilon = \tau_1 + \epsilon(\tau_2 - \tau_1)$, with $\epsilon \neq 1/2$ (see the discussion in Sec. 3.3 for the original definition of $\bar{\tau}$). Here ϵ could be a constant, or it could be a function of σ and of the angular coordinates θ, ϕ. Such a convention can be defined from the same minimal set of elements that we just discussed, plus the arbitrary scalar field $\epsilon(\sigma, \theta, \phi)$. What distinguishes the Reichenbachized convention from plain Märzke–Wheeler synchrony? Why should we prefer the latter, if both yield noninertial time coordinates?

Unfortunately, when we extended our attention to accelerated observers we effectively walked out of special relativity, so we cannot use the arguments of Sec. 4.2.4, which show that standard simultaneity is intimately linked with fundamental aspects of special relativity. Neither can we appeal to Malament's theorem: none of the symmetries that preserve the causal structure of spacetime preserves a generic accelerated worldline. In Malament's language, the only $\mathcal{P}(\tau)$ causal automorphism is the identity: so the Märzke–Wheeler simultaneity slices cannot be obtained from the causal automorphisms, and the proof collapses.

Nevertheless, we still have some grounds for rejecting Reichenbach–Märzke–Wheeler synchrony. It might be possible to show that a particular choice of the $\epsilon(\sigma, \theta, \phi)$ field serves a specific explanatory purpose: for instance, there could be a choice of $\epsilon(\sigma, \theta, \phi)$ under which Reichenbach–Märzke–Wheeler coordinates satisfy some desirable properties on top of the ones outlined at the beginning of Sec. 4.3. If this is not the case, then $\epsilon(\sigma, \theta, \phi)$ is just a spurious absolute object which should have no business in the definition of synchrony. So far, we have not been able to think of any especially useful definition of $\epsilon(\sigma, \theta, \phi)$.

What about a constant ϵ? Surely a single real number is inoffensive enough, even if it is an absolute object! Maybe not. One of the *desirable properties* of Märzke–Wheeler coordinates was that they reduce to Lorentz coordinates in a neighborhood of the observer's trajectory (a neighborhood small enough that the trajectory looks approximately inertial). If we want this property to survive, then ϵ must be $1/2$, which gives the plain Märzke–Wheeler simultaneity. Indeed, this property *should* survive, because in essence it says that accelerated simultaneity reduces to the nonconventional simultaneity of special relativity where the effects of acceleration are small enough to be neglected.

Ultimately, it appears that our discussion brought us to a tentative verdict of *nonconventionality* even for the *extension* of Einstein synchronization to accelerated observers.

Appendix A

Derivation of a Constant Homogeneous Flat Metric

We reproduce here Rohrlich's derivation (1963) of the metric appropriate to a constant homogeneous gravitational field. We begin from the most general Lorentzian metric $g_{\mu\nu}$. Staticity implies that the time coordinate can be separated and the metric written as

$$ds^2 = -g_{tt}\, dt^2 + g_{ij}\, dx^i dx^j; \tag{A.1}$$

where no coefficient depends on t. We can now diagonalize the spatial metric and impose homogeneity along coordinates x and y: thus all coefficients will be functions of z only,

$$ds^2 = -D_t(z)\, dt^2 + D_x(z)\, dx^2 + D_y(z)\, dy^2 + D_z(z)\, dz^2. \tag{A.2}$$

The only nonvanishing Christoffel coefficients turn out to be

$$\begin{gathered}
\Gamma^t{}_{tz} = \frac{1}{2}\frac{D_t'(z)}{D_t(z)}, \quad \Gamma^x{}_{xz} = \frac{1}{2}\frac{D_x'(z)}{D_x(z)}, \\
\Gamma^z{}_{xx} = -\frac{1}{2}\frac{D_x'(z)}{D_z(z)}, \quad \Gamma^z{}_{zz} = \frac{1}{2}\frac{D_z'(z)}{D_z(z)}, \quad \Gamma^z{}_{tt} = \frac{1}{2}\frac{D_t'(z)}{D_z(z)}.
\end{gathered} \tag{A.3}$$

We require flatness by imposing that all components of the Riemann tensor vanish. Thus we get the set of equations

$$D_t' D_x' = D_t' D_y' = D_x' D_y' = 0; \tag{A.4}$$

$$2D_i'' - \frac{(D_i')^2}{D_i} - \frac{D_z' D_i'}{D_z} = 0 \quad \text{for } i = x, y, t; \tag{A.5}$$

which imply that two out of D_t, D_x and D_y must be constant. We now impose the Newtonian limit. The equation of motion for test particles falling in the gravitational field is the geodesic equation:

$$\frac{d^2 x^\rho}{d\tau^2} + \Gamma^\mu{}_{\mu\nu} u^\mu u^\nu = 0. \tag{A.6}$$

For motions much slower than the speed of light we may approximate the proper time τ with t, and the four-velocity $dx^\mu/d\tau$ with $(1,0,0,0)$. For the vertical component of the motion, we get

$$\frac{d^2z}{dt^2} + \Gamma^z{}_{00} = \frac{d^2z}{dt^2} + \frac{D_t'(z)}{2D_z(z)} = 0. \tag{A.7}$$

From this equation, we learn that D_t cannot be a constant, because otherwise we would not obtain a gravitational force field in the nonrelativistic limit. Thus D_x and D_y must be constants that we can absorb in the definition of x and y. Our new form for the metric is then

$$ds^2 = -D_t(z)\,dt^2 + dx^2 + dy^2 + D_z(z)\,dz^2, \tag{A.8}$$

where by Eq. (A.5),

$$\frac{2D_t''}{D_t'} - \frac{D_t'}{D_t} = \frac{D_z'}{D_z}; \tag{A.9}$$

hence,

$$D_z(z) = \left(C\frac{d}{dz}\sqrt{D_t(z)}\right)^2, \tag{A.10}$$

where C is a constant of integration. By Eq. (A.7), for small displacements z, we get $C = 1/g$, and

$$D_t(z) \to 1 + 2gz \quad \text{for } gz \ll 1, gt \ll 1. \tag{A.11}$$

We may therefore write our line element in the final form

$$ds^2 = -D_t(z)\,dt^2 + dx^2 + dy^2 + (\sqrt{D_t(z)}'/g)^2 dz^2, \tag{A.12}$$

where $D_t(z)$ is required to approximate $1 + 2gz$ to first order in gz. Finally we rewrite the updated nonzero Christoffel coefficients,

$$\Gamma^t{}_{tz} = \frac{\sqrt{D_t(z)}'}{\sqrt{D_t(z)}}, \quad \Gamma^z{}_{zz} = \frac{\sqrt{D_t(z)}''}{\sqrt{D_t(z)}'}, \quad \Gamma^z{}_{tt} = g^2\frac{\sqrt{D_t(z)}}{\sqrt{D_t(z)}'}. \tag{A.13}$$

Appendix B

Stationary Trajectories in Flat Spacetime (Synge's Helixes)

Synge (1967) solved the *relativistic Frenet–Serret equations*,

$$\begin{cases} \dot{u}^\mu &= c_1 n_1^\mu, \\ \dot{n}_1^\mu &= c_2 n_2^\mu + c_1 u^\mu, \\ \dot{n}_2^\mu &= c_3 n_3^\mu - c_2 n_1^\mu, \\ \dot{n}_3^\mu &= -c_3 n_2^\mu \end{cases} \tag{B.1}$$

(where u^μ is the four-velocity and n_i^μ are the three *normals*), by setting the curvature coefficients c_1, c_2, and c_3 to constants. We now briefly summarize Synge's classification of the resulting trajectories (for pedagogical purposes, we invert Synge's enumeration). In Fig. B.1, we show examples of these curves.

Inertial worldlines (type IV)

All curvatures vanish.

Hyperbolic motion (type III)

(*Hyper-Stacy*) $c_2 = c_3 = 0$. The only nonzero curvature is the acceleration. Motion is restricted to a (1 + 1)-dimensional hyperplane; the trajectory is spatially unlimited and the three-velocity approaches asymptotically the speed of light. In a suitable Lorentz frame, we can write the worldline as

$$\begin{cases} t = c_1^{-1} \sinh c_1 \tau, \\ x = c_1^{-1} \cosh c_1 \tau, \quad \text{(Type III)} \\ y = z = 0, \end{cases} \tag{B.2}$$

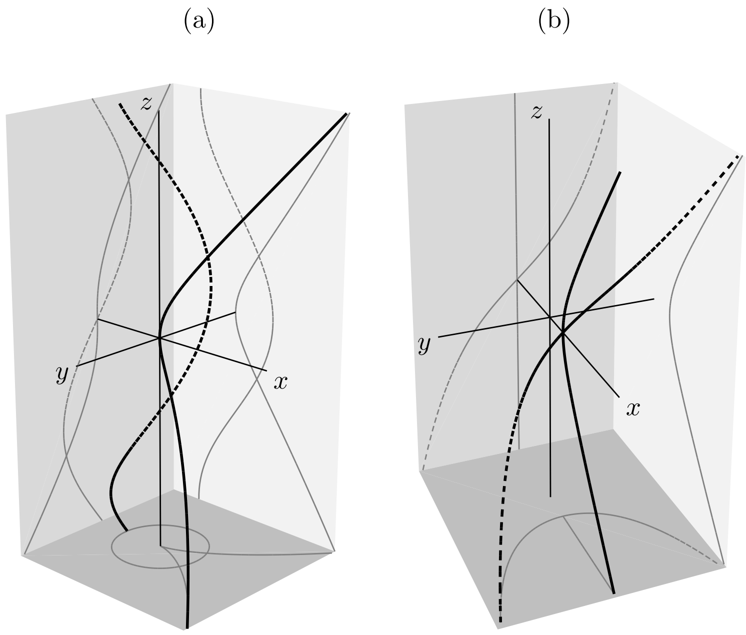

Figure B.1: Synge's helixes. (a): Type-IIc (shown dashed), and type-IIb helixes. (b): Type-IIa (dashed), and type-III helixes. Notice the cusp in the xy-plane projection of the type-IIb curve; also notice that the type-III helix coincides with projection of the type-IIa curve on the xz plane.

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where τ is proper time, and c_1 is the magnitude of the acceleration.

Plane helixes (type II)

Only $c_3 = 0$: the spatial curvature c_2 allows nontrivial motion in a $(2+1)$-dimensional hyperplane. There are three subtypes.

Uniform circular motion (Roto-Stacy, type IIc)

If $c_2^2 - c_1^2 > 0$, the worldline winds up in a spatially limited domain. It is a circular helix of radius $c_1/(c_2^2 - c_1^2)$ and angular velocity $\sqrt{c_2^2 - c_1^2}$.

$$\begin{cases} t = \dfrac{c_2}{c_2^2 - c_1^2} \sqrt{c_2^2 - c_1^2}\,\tau, \\ x = \dfrac{c_1}{c_2^2 - c_1^2} \cos \sqrt{c_2^2 - c_1^2}\,\tau, \\ y = \dfrac{c_1}{c_2^2 - c_1^2} \sin \sqrt{c_2^2 - c_1^2}\,\tau, \\ z = 0. \end{cases} \qquad \text{(Type IIc)} \qquad \text{(B.3)}$$

Cusped motion (type IIb)

If $c_1^2 - c_2^2 = 0$, the result is a runaway curve (although it approaches spatial infinity only cubically in time, rather than exponentially as type III), with a peculiar cusp.

$$\begin{cases} t = \tau + \frac{1}{6}\, c_1^2 \tau^3, \\ x = \frac{1}{2}\, c_1 \tau^2, \\ y = \frac{1}{6}\, c_1^2 \tau^3 \\ z = 0. \end{cases} \qquad \text{(Type IIb)} \tag{B.4}$$

Skewed hyperbolic motion (type IIa)

If $c_1^2 - c_2^2 > 0$, the spatial curvature c_2 is not strong enough to wind up the worldline, which becomes spatially unlimited and approaches asymptotically the speed of light. In fact, this solution may be considered as a type-III helix combined with a linear, uniform motion.

$$\begin{cases} t = \frac{c_1}{c_1^2 - c_2^2} \sinh \sqrt{c_1^2 - c_2^2}\, \tau, \\ x = \frac{c_1}{c_1^2 - c_2^2} \cosh \sqrt{c_1^2 - c_2^2}\, \tau, \\ y = \frac{c_2}{c_1^2 - c_2^2} \sqrt{c_1^2 - c_2^2}\, \tau, \\ z = 0. \end{cases} \qquad \text{(Type IIa)} \tag{B.5}$$

General case (type I)

All curvatures have a finite value, and the trajectory is truly four-dimensional. The resulting helix is a product (of sorts) between a type-III and a type-IIc motion, each of which takes place in a two-dimensional hyperplane.

$$\begin{cases} t = r\chi^{-1} \sinh \chi\, \tau, \\ x = q\gamma^{-1} \sin \gamma\, \tau, \\ y = q\gamma^{-1} \cos \gamma\, \tau, \\ z = r\chi^{-1} \cosh \chi\, \tau, \end{cases} \qquad \text{(Type I)} \tag{B.6}$$

where

$$\begin{cases} \chi^2 = (c_1^2 - c_2^2 - c_3^2 + R)/2, \\ \gamma^2 = (-c_1^2 + c_2^2 + c_3^2 + R)/2, \\ r^2 = [(c_1^2 + c_2^2 + c_3^2)/R + 1]/2, \\ q^2 = [(c_1^2 + c_2^2 + c_3^2)/R - 1]/2, \\ R^2 = (c_1^2 - c_2^2 - c_3^2)^2 + 4c_1^2 c_3^2. \end{cases} \tag{B.7}$$

Appendix C

Märzke–Wheeler Coordinates for Uniformly Rotating Observers

Roto-Stacy's worldline, Eq. (3.13), is given in Cartesian coordinates by

$$\begin{cases} t &= \sqrt{1+R^2\Omega^2}\,\tau, \\ x &= R\cos\Omega\tau, \\ y &= R\sin\Omega\tau. \end{cases} \qquad (\textit{Roto-Stacy}\text{: worldline}) \tag{C.1}$$

We seek equations for the surface $\Sigma_{\bar{\tau}=0}$, which is generated by the concentric curves $S(\sigma)$ of constant σ; each curve $S(\sigma)$ is defined as the intersection of the future lightcone of $\mathcal{P}(-\sigma)$ with the past lightcone of $\mathcal{P}(\sigma)$ (see Fig. C.1).

A point $\mathcal{Q}$ belongs to the future lightcone of $\mathcal{P}(-\sigma)$ if the spatial distance between $\mathcal{P}(-\sigma)$ and $\mathcal{Q}$ equals the coordinate-time difference between them; that is, if

$$\left|\mathbf{x}[\mathcal{Q}] - \mathbf{x}[\mathcal{P}(-\sigma)]\right| = t[\mathcal{Q}] - t[\mathcal{P}(-\sigma)] = \Delta t(-\sigma); \tag{C.2}$$

a similar relation is true for points on the past lightcone of $\mathcal{P}(\sigma)$:

$$\left|\mathbf{x}[\mathcal{Q}] - \mathbf{x}[\mathcal{P}(\sigma)]\right| = t[\mathcal{P}(\sigma)] - t[\mathcal{Q}] = \Delta t(\sigma). \tag{C.3}$$

Summing the two equations, we get

$$\begin{aligned} &\left|\mathbf{x}[\mathcal{Q}] - \mathbf{x}[\mathcal{P}(-\sigma)]\right| + \left|\mathbf{x}[\mathcal{Q}] - \mathbf{x}[\mathcal{P}(\sigma)]\right| = \\ &\qquad = t[\mathcal{P}(\sigma)] - t[\mathcal{P}(-\sigma)] = \Delta t(\sigma) + \Delta t(-\sigma) = 2\sqrt{1+R^2\Omega^2}\sigma; \end{aligned} \tag{C.4}$$

that is, the points on $S(\sigma)$ describe an ellipse in the spatial plane. These ellipses have $\mathcal{P}(-\sigma)$ and $\mathcal{P}(\sigma)$ as their foci, and their centers $\mathcal{C}(\sigma)$ are at

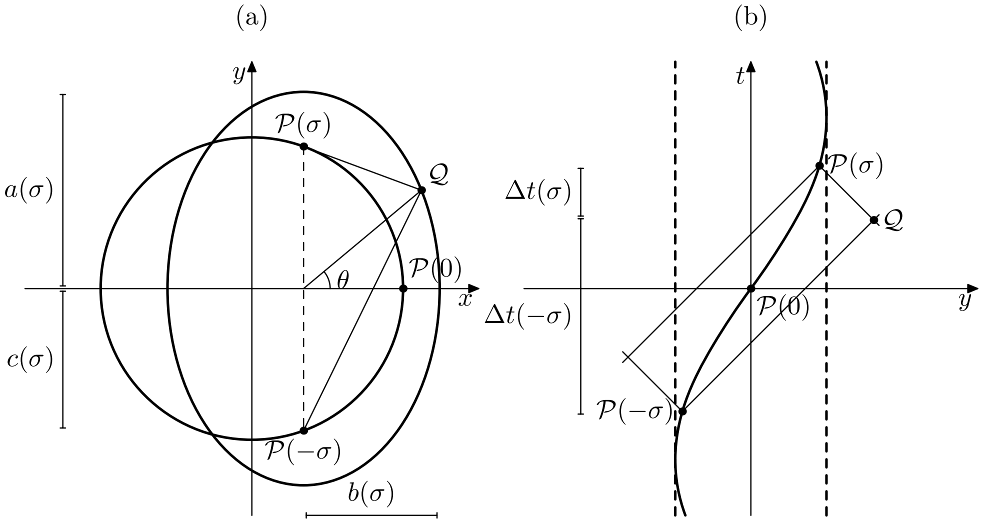

Figure C.1: Geometric construction of Märzke–Wheeler constant-σ surfaces for *Roto-Stacy.* Her worldline's projection is (a) a circle in the xy plane; (b) a sinusoidal curve in the yt plane.

($x = R\cos\Omega\sigma$, $y = 0$). We parametrize the ellipses in the obvious way,

$$\begin{cases} x &= b(\sigma)\cos\theta + R\cos\Omega\sigma, \\ y &= a(\sigma)\sin\theta. \end{cases} \qquad \text{(ellipses } S(\sigma)\text{)} \tag{C.5}$$

The length $a(\sigma)$ of the major semiaxis is given by the half-sum of the distances between any point on $S(\sigma)$ and the two foci:

$$a(\sigma) = \frac{1}{2}\Big\{\big|\mathbf{x}[\mathcal{Q}] - \mathbf{x}[\mathcal{P}(-\sigma)]\big| + \big|\mathbf{x}[\mathcal{Q}] - \mathbf{x}[\mathcal{P}(\sigma)]\big|\Big\} = \sqrt{1 + R^2\Omega^2}\,\sigma; \tag{C.6}$$

also, from Eq. (C.1) the half-distance between the foci is $c(\sigma) = R\sin\Omega\sigma$, so we find the length of the minor semiaxis $b(\sigma)$ as

$$b(\sigma) = \sqrt{a^2(\sigma) - c^2(\sigma)} = \sqrt{(1 + R^2\Omega^2)\sigma^2 - R^2\sin^2\Omega\sigma}. \tag{C.7}$$

To complete our characterization of $\Sigma_{\bar{\tau}=0}$, we need only the coordinate time of the points on the curves $S(\sigma)$. From Fig. C.1b we have

$$\begin{aligned} t[\mathcal{Q}] = t[\mathcal{P}(\sigma)] - \Delta t(\sigma) &= \frac{1}{2}\big\{\Delta t(-\sigma) + \Delta t(\sigma)\big\} - \Delta t(\sigma) = \\ &= \frac{1}{2}\big|\mathbf{x}[\mathcal{Q}] - \mathbf{x}[\mathcal{P}(-\sigma)]\big| - \frac{1}{2}\big|\mathbf{x}[\mathcal{Q}] - \mathbf{x}[\mathcal{P}(\sigma)]\big|; \end{aligned} \tag{C.8}$$

then by explicit calculation we find that $t[\mathcal{Q}] = c(\sigma)\sin\theta$, using Eqs. (C.5)–(C.7). Altogether we obtain the surface described by Eq. (3.16) and shown in Fig. 3.3. Given the symmetry of *Roto-Stacy*'s motion, the effect of moving from $\Sigma_{\bar{\tau}=0}$ to $\Sigma_{\bar{\tau}=\Delta\bar{\tau}}$ will be just a translation in t by $\sqrt{1+R^2\Omega^2}\,\Delta\bar{\tau}$, together with a rotation of x and y by an angle $\Omega\,\Delta\bar{\tau}$; the complete transformation between Märzke–Wheeler and Lorentz coordinates is therefore that given in Eq. (3.17).

Appendix D

Märzke–Wheeler Coordinates for the Paradox of the Twins: Linear Motion

For simplicity, we use Penelope's Lorentz coordinates to parametrize Ulysses' worldline (shown in Fig. 3.4), placing the origin $(0,0)$ in $\mathcal{C}_U$, so that the worldline is described by $x = -v|t|$. Proceeding as in App. C, we see that the Märzke–Wheeler constant-time surface that is simultaneous to $\mathcal{P}(t_0)$ is given by the events $\mathcal{Q}$ such that, for some s,

$$\begin{cases} \big|x[\mathcal{Q}] - x[\mathcal{P}(t_0 - s)]\big| & = t[\mathcal{Q}] - t[\mathcal{P}(t_0 - s)], \\ \big|x[\mathcal{Q}] - x[\mathcal{P}(t_0 + s)]\big| & = t[\mathcal{P}(t_0 + s)] - t[\mathcal{Q}]. \end{cases} \tag{D.1}$$

We simplify our notation by setting $t = t[\mathcal{Q}]$ and $x = x[\mathcal{Q}]$, and we insert the explicit form of Ulysses' worldline into Eq. (D.1):

$$\begin{cases} \big|x + v|t_0 - s|\big| & = t - (t_0 - s), \\ \big|x + v|t_0 + s|\big| & = (t_0 + s) - t. \end{cases} \tag{D.2}$$

If we are concerned only with events to the left of Ulysses' trajectory, the outer absolute values can be exchanged for a minus. Summing and subtracting the equations, we obtain the following expressions for x and t:

$$\begin{cases} -x & = s \quad + \frac{1}{2}\big(v|t_0 - s| + v|t_0 + s|\big), \\ \;\; t & = t_0 \quad + \frac{1}{2}\big(v|t_0 + s| - v|t_0 - s|\big). \end{cases} \quad \text{(events simultaneous to } \mathcal{P}(t_0)) \tag{D.3}$$

Let us take $t_0 > 0$, and examine Eq. (D.3): if an event $\mathcal{Q}$, simultaneous to $\mathcal{P}(t_0)$, belongs to region E of Fig. 3.4c, both $\mathcal{P}(t_0 - s)$ and $\mathcal{P}(t_0 + s)$ will be in region E. It follows that $t_0 - s > 0$ and $t_0 + s > 0$, and therefore

$$\begin{cases} -x &= s \quad + v t_0, \\ \;\; t &= t_0 \quad + v s. \end{cases} \qquad \text{(region } E\text{)} \tag{D.4}$$

In a neighborhood of Ulysses' worldline, these equations reproduce the slices of his constant Lorentz time. On the other hand, if Q belongs to region C, then $t_0 - s < 0$, $t_0 + s > 0$, and

$$\begin{cases} -x &= (1+v)s, \\ \;\; t &= (1+v)t_0. \end{cases} \qquad \text{(region } C\text{)} \tag{D.5}$$

These relations create the flat structure of Märzke–Wheeler slices shown in Fig. 3.4c. The two coordinate patches of Eqs. (D.4), (D.5) join correctly on $x = -|t|$, where $s = t_0$.

Appendix E

Märzke–Wheeler Coordinates for the Paradox of the Twins: Circular Motion

In this scenario, we make the twins start together at the event $\mathcal{F}$ with Lorentz coordinates $t = 0$, $x = R$, and $y = 0$. While the stationary twin Penelope stands fixed in space, Ulysses completes one circular orbit according to Eqs. (3.13) and (C.1), and rejoins Penelope at the event $\mathcal{G}$, defined by $t = 2\pi\Omega^{-1}\sqrt{1 + \Omega^2 R^2}$, $x = R$, and $y = 0$. After one revolution, Ulysses' proper-time lapse is $\Delta\tau = 2\pi\Omega^{-1}$; Penelope's proper time coincides with the Lorentz coordinate time, so that her proper-time lapse is $\sqrt{1 + \Omega^2 R^2}$ times Ulysses'. It turns out that this coefficient is just $\gamma = (1 - v^2)^{-1/2}$, because Ulysses moves with a constant velocity $v = \Omega R/\sqrt{1 + \Omega^2 R^2}$. In the end, we get the same differential aging of the twins as in the simpler linear geometry of App. D, and also as predicted by a naïve application of the time dilation rule.

To study the local distribution of this differential aging, we need to determine the Märzke–Wheeler time (according to Ulysses) of all the events on Penelope's worldline. It is expedient to work in Lorentz polar coordinates centered around Penelope's location. Then Ulysses' worldline is given by

$$\begin{cases} t &= \sqrt{1 + R^2\Omega^2}\,\tau, \\ \rho &= 2R\sin\dfrac{\Omega\tau}{2}, \\ \theta &= \dfrac{\pi}{2} - \dfrac{\Omega\tau}{2}. \end{cases} \qquad (\textit{Roto-Ulysses}\text{: worldline}) \qquad \text{(E.1)}$$

Let us now proceed in analogy with App. D. Eliminating the parameter τ, we describe Ulysses' worldline as

$$\mathcal{P}(t) : \rho = 2R \sin\left(\frac{\Omega t}{2\sqrt{1+R^2\Omega^2}}\right). \quad (\textit{Roto-Ulysses}\text{: worldline}) \tag{E.2}$$

If we take only target events $\mathcal{Q}$ that are on Penelope's worldline, the lightcone conditions (D.1) can be restated simply as

$$\begin{cases} \rho[\mathcal{P}(t_0 - s)] & = t[\mathcal{Q}] - t[\mathcal{P}(t_0 - s)], \\ \rho[\mathcal{P}(t_0 + s)] & = t[\mathcal{P}(t_0 + s)] - t[\mathcal{Q}], \end{cases} \tag{E.3}$$

where t_0 identifies an event along Ulysses' worldline, and t and s identify the simultaneous event (in the Märzke–Wheeler sense) along Penelope's worldline. Now, set $t = t[\mathcal{Q}]$ and use Eq. (E.2):

$$\begin{cases} 2R \sin\left(\dfrac{\Omega(t_0 - s)}{2\sqrt{1+\Omega^2 R^2}}\right) = t - t_0 + s, \\ 2R \sin\left(\dfrac{\Omega(t_0 + s)}{2\sqrt{1+\Omega^2 R^2}}\right) = t_0 + s - t. \end{cases} \tag{E.4}$$

We sum and subtract these two equations, and rearrange their terms:

$$\begin{cases} t = t_0 - 2R \sin\left(\dfrac{\Omega s}{2\sqrt{1+\Omega^2 R^2}}\right) \cos\left(\dfrac{\Omega t_0}{2\sqrt{1+\Omega^2 R^2}}\right), \\ s = 2R \sin\left(\dfrac{\Omega t_0}{2\sqrt{1+\Omega^2 R^2}}\right) \cos\left(\dfrac{\Omega s}{2\sqrt{1+\Omega^2 R^2}}\right). \end{cases} \tag{E.5}$$

These new equations must be solved together for t and s as functions of t_0. The resulting distribution for differential aging is shown in Fig. E.1, and it is a smoother version of the distribution that we obtained for the linear geometry of App. D (see Fig. 3.5). Interestingly, if we set

$$\{\tilde{t}, \tilde{s}, \tilde{t}_0\} = \frac{\Omega}{\sqrt{1+\Omega^2 R^2}} \{t, s, t_0\}, \tag{E.6}$$

and then multiply Eqs. (E.5) by $\Omega/\sqrt{1+\Omega^2 R^2}$, we find that the solutions $\tilde{t}(\tilde{t}_0)$ and $\tilde{s}(\tilde{t}_0)$ depend on the product ΩR, but not on Ω and R separately. This means that in these units, where the total elapsed Lorentz time is just 2π, the shape of curve that describes the aging distribution depends on Ulysses' absolute velocity ($\Omega R = v/\sqrt{1-v^2}$), but not on the radius and angular frequency of his helix.

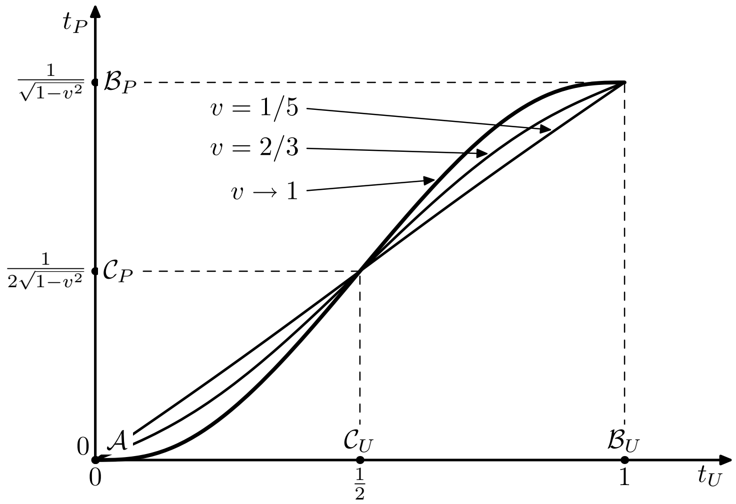

Figure E.1: Circular version of the paradox of the twins.

The graph shows Penelope's proper time, in units of Ulysses' total proper-time lapse, as determined by Ulysses with Märzke–Wheeler slicing. In these renormalized units, the shape of the curve depends only on Ulysses' velocity. For $v \to 0$, the curve tend to a straight line; for $v \to 1$, to a limit curve.

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